SWISS CONTRIBUTION

NANOSPIN

Second harmonic response to spin transfer torque in metallic spin valves

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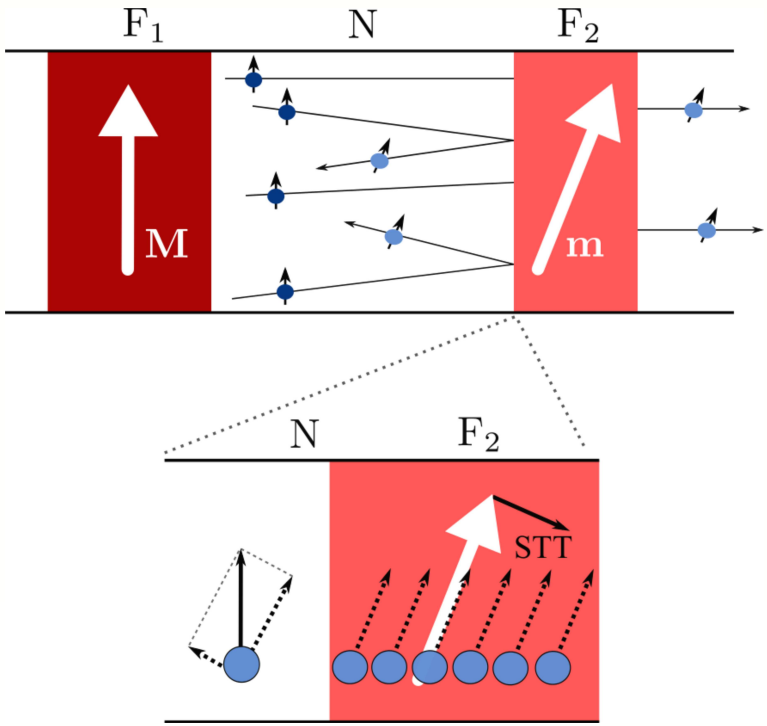
AGH

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

IFM PAN

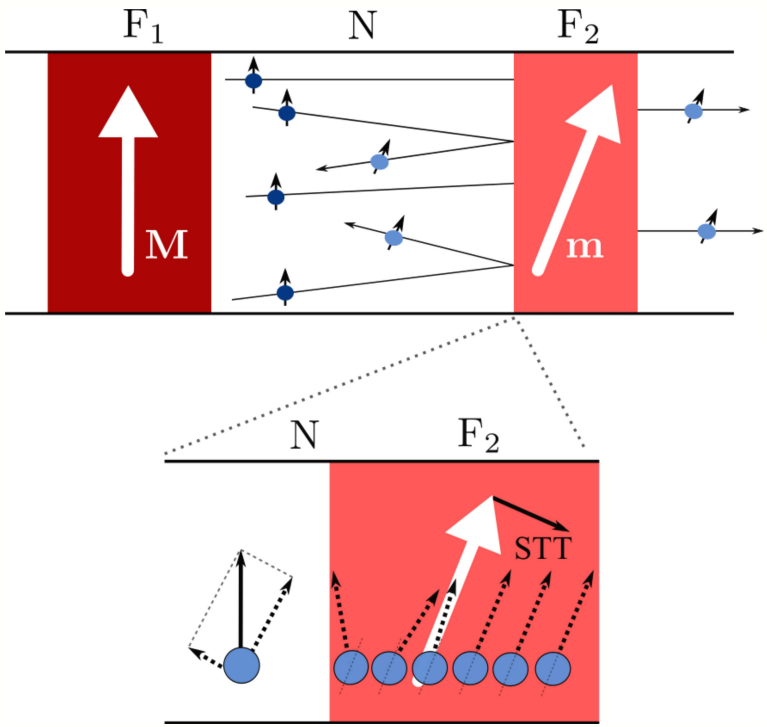
Outline

Transverse spin diffusion in metallic spin valves



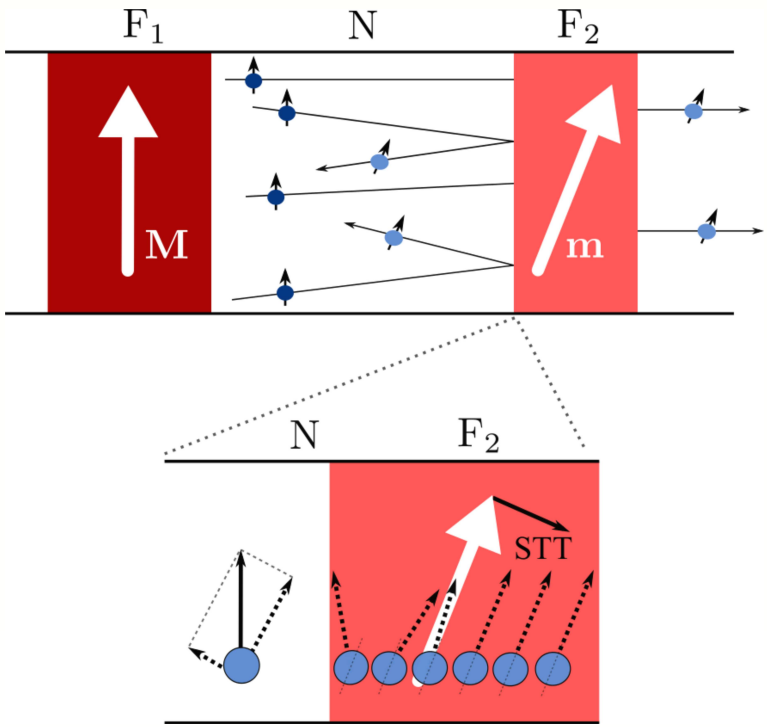
Transverse components disappear at the N/F interface.

Transverse spin diffusion in metallic spin valves



Transverse components comes into the ferromagnet and spin accumulation *precess* and *relaxes* around the local exchange field.

Transverse spin diffusion in metallic spin valves



Transverse components comes into the ferromagnet and spin accumulation *precess* and *relaxes* around the local exchange field.

Continuity equation

$$\frac{\mathrm{d}\check{f}}{\mathrm{d}t} - \frac{\partial \check{\mathrm{æ}}}{\partial x} = \frac{1}{\tau_{\mathrm{sf}}}\left[\check{f} - \check{1}\frac{\mathrm{Tr}\,\check{f}}{2}\right]$$

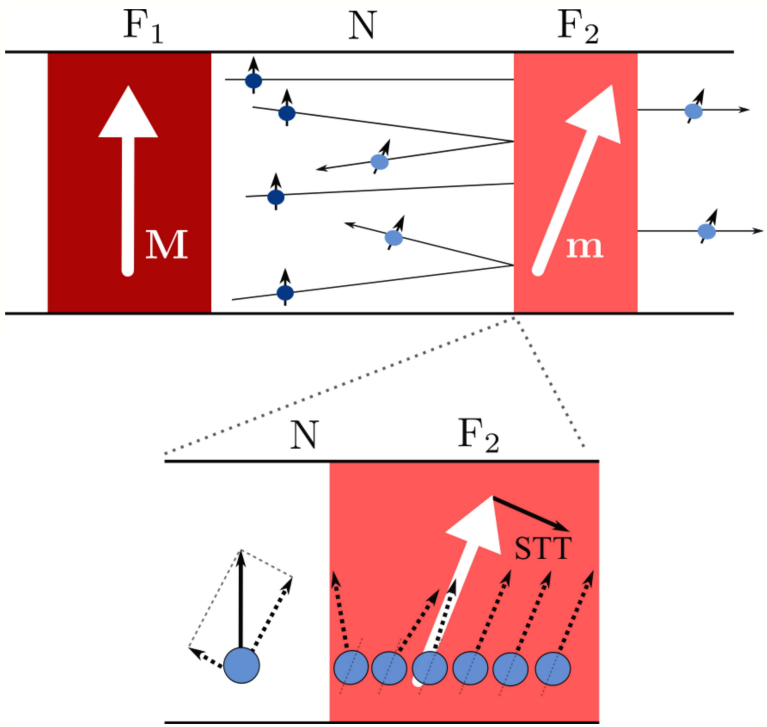
Particle-spin distribution function

$$\check{f} = f_0\,\check{1} + \boldsymbol{f}\cdot\boldsymbol{\sigma}$$

Particle-spin current density

$$\check{\mathrm{æ}} = \frac{1}{2}\left(j_0\check{1} + \boldsymbol{j}\cdot\boldsymbol{\sigma}\right)$$

Transverse spin diffusion in metallic spin valves



Transverse components comes into the ferromagnet and spin accumulation *precess* and *relaxes* around the local exchange field.

In magnetic film

$$\mathcal{H}_{\mathrm{s-d}} = -\frac{J}{2}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{S}}$$

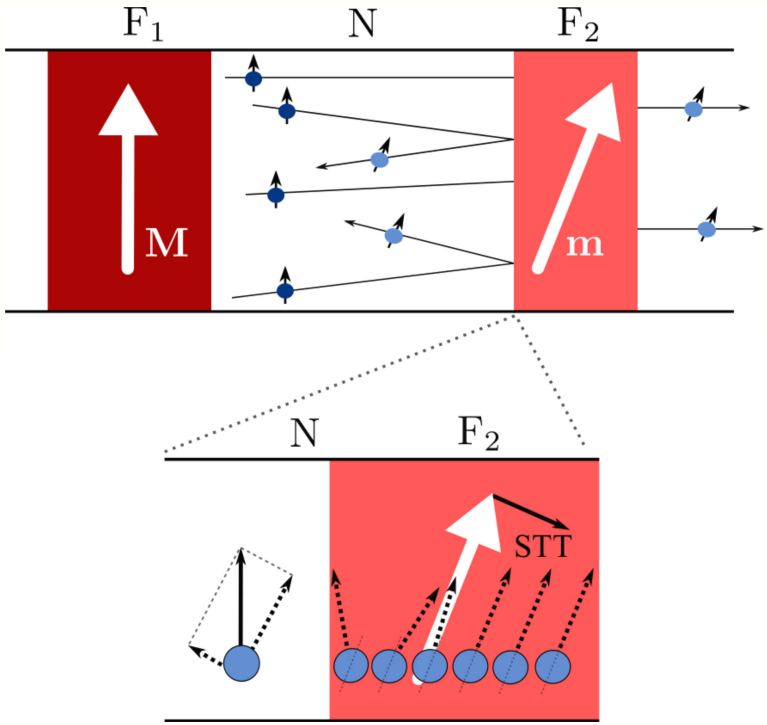
where J is the s-d coupling constant

$$\frac{\mathrm{d}\check{f}}{\mathrm{d}t} = \frac{i}{\hbar}\left[\mathcal{H}_{\mathrm{s-d}}, \check{f}\right]$$

which leads to

$$\frac{\mathrm{d}\check{f}}{\mathrm{d}t} = \frac{J}{\hbar}\left(\hat{\boldsymbol{S}}\times\boldsymbol{f}\right)\cdot\boldsymbol{\sigma}$$

Transverse spin diffusion in metallic spin valves



Transverse components comes into the ferromagnet and spin accumulation *precess* and *relaxes* around the local exchange field.

Continuity equation

$$\frac{\mathrm{d}f_0}{\mathrm{d}t} = \frac{\partial j_0}{\partial x}$$

$$\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}t} = \frac{J}{\hbar}\left(\hat{\mathbf{S}} \times \mathbf{f}\right) + \frac{\partial \mathbf{j}}{\partial x} + \frac{\mathbf{f}}{\tau_{\mathrm{sf}}}$$

- Local net spin moment: $\hat{\mathbf{S}} = (0, 0, 1)$
- Spin distribution: $\mathbf{f} = (f_x, f_y, f_z)$
- Spin current: $\mathbf{j} = (j_x, j_y, j_z)$
- s-d exchange parameter: J
- Spin-flip time: τ_{sf}
- Particle current: j_0
- Particle distribution: f_0

Outline

Spin diffusion

Diffusion equation

$$\check{\jmath} = \begin{pmatrix} j_\uparrow & j_{\uparrow\downarrow} \\ j_{\downarrow\uparrow} & j_\downarrow \end{pmatrix} = -\begin{pmatrix} D_\uparrow \frac{\partial f_\uparrow}{\partial x} & D_\perp \frac{\partial f_{\uparrow\downarrow}}{\partial x} \\ D_\perp \frac{\partial f_{\downarrow\uparrow}}{\partial x} & D_\downarrow \frac{\partial f_\downarrow}{\partial x} \end{pmatrix}$$

where $j_{\uparrow(\downarrow)} = j_0 \pm j_z$, $j_{\uparrow\downarrow} = j_x - ij_y$, and $j_{\downarrow\uparrow} = (j_{\uparrow\downarrow})^*$
$f_{\uparrow(\downarrow)} = f_0 \pm f_z$, $f_{\uparrow\downarrow} = fx - if_y$ and $f_{\downarrow\uparrow} = (f_{\uparrow\downarrow})^*$
$D_\uparrow$, $D_\downarrow$, and $D_\perp$ are diffusion constants

from the free electrons model $D_{\uparrow(\downarrow)} = D_0/(1 \mp \beta)$ with $D_0 = \dfrac{\pi^2\hbar^3}{\sqrt{2}\rho^* e^2 m_e^{3/2} \epsilon_F^{1/2}}$

and $D_\perp = (D_\uparrow + D_\downarrow)/2$

Spin diffusion

Diffusion equation

$$\check{\jmath} = \begin{pmatrix} j_\uparrow & j_{\uparrow\downarrow} \\ j_{\downarrow\uparrow} & j_\downarrow \end{pmatrix} = - \begin{pmatrix} D_\uparrow \frac{\partial f_\uparrow}{\partial x} & D_\perp \frac{\partial f_{\uparrow\downarrow}}{\partial x} \\ D_\perp \frac{\partial f_{\downarrow\uparrow}}{\partial x} & D_\downarrow \frac{\partial f_\downarrow}{\partial x} \end{pmatrix}$$

Longitudinal components

$$j_0 = -\frac{2D_0}{1-\beta^2}\left(\frac{\partial f_0}{\partial x} + \beta \frac{\partial f_z}{\partial x}\right)$$

$$j_z = -\frac{2D_0}{1-\beta^2}\left(\beta \frac{\partial f_0}{\partial x} + \frac{\partial f_z}{\partial x}\right)$$

Transverse components

$$j_x = -2D_\perp \frac{\partial f_x}{\partial x}$$

$$j_y = -2D_\perp \frac{\partial f_y}{\partial x}$$

Longitudinal spin transport

Diffusion equations

$$\frac{\partial^2 f_z}{\partial x^2} = \frac{f_z}{\lambda_{sf}^2} \qquad \text{and} \qquad \frac{\partial^2 f_0}{\partial x^2} = -\beta \frac{\partial^2 f_z}{\partial x^2}$$

Longitudinal charge and spin accumulation

$$\mu_z(x) = A\, e^{x/\lambda_{sf}} + B\, e^{-x/\lambda_{sf}}$$
$$\mu_0(x) = -\beta\, \mu_z(x) + Cx + G$$

Longitudinal charge and spin current

$$\frac{1}{\rho(\epsilon_F)} j_0 = -2\, C \frac{D_0}{1-\beta^2}$$
$$\frac{1}{\rho(\epsilon_F)} j_z(x) = \frac{\beta}{\rho(\epsilon_F)} j_0 - \frac{2D_0}{\lambda_{sf}} \left[A\, e^{x/\lambda_{sf}} - B\, e^{-x/\lambda_{sf}} \right]$$

Transverse spin transport

Diffusion equations

$$\frac{\partial^2 f_x}{\partial x^2} = -\frac{f_y}{\lambda_J^2} + \frac{f_x}{\lambda_\perp^2}$$

$$\frac{\partial^2 f_y}{\partial x^2} = \frac{f_x}{\lambda_J^2} + \frac{f_y}{\lambda_\perp^2}$$

General solution for f_x

$$f_x = C_1\, e^{x/l_-} + C_2\, e^{-x/l_-} + C_3\, e^{x/l_+} + C_4\, e^{-x/l_+}$$

where $\frac{1}{l_\pm} = \sqrt{\frac{1}{\lambda_\perp^2} \pm i\frac{1}{\lambda_J^2}} = \frac{1}{\lambda_t} \pm i\frac{1}{\lambda_p}$ where $\frac{1}{\lambda_{t(p)}} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\lambda_J^2} \pm \frac{1}{\lambda_\perp^2}}$

- λ_t: transverse decay length
- λ_p precessional length

we define $\Delta = \lambda_t/\lambda_p$, which is $\Delta < 1$

$$\frac{1}{l_\pm} = \frac{1 \pm i\,\Delta}{\lambda_t}$$

Transverse spin transport

Diffusion equations

$$\frac{\partial^2 f_x}{\partial x^2} = -\frac{f_y}{\lambda_J^2} + \frac{f_x}{\lambda_\perp^2}$$

$$\frac{\partial^2 f_y}{\partial x^2} = \frac{f_x}{\lambda_J^2} + \frac{f_y}{\lambda_\perp^2}$$

Transverse spin accumulation

$$\mu_{x(y)} = \left[A_1 \cos\left(\frac{\Delta x}{\lambda_t}\right) \pm B_1 \sin\left(\frac{\Delta x}{\lambda_t}\right)\right] e^{x/\lambda_t} \pm \left[A_2 \cos\left(\frac{\Delta x}{\lambda_t}\right) \pm B_2 \sin\left(\frac{\Delta x}{\lambda_t}\right)\right] e^{-x/\lambda_t}$$

Transverse spin current

$$\frac{1}{\rho(\epsilon_F)} j_{x(y)} = \pm\frac{D_\perp}{\lambda_t} \left\{ \left[(\Delta A_1 - B_1) \sin\left(\frac{\Delta x}{\lambda_t}\right) \mp (A_1 + \Delta B_1) \cos\left(\frac{\Delta x}{\lambda_t}\right)\right] e^{x/\lambda_t} \pm \right.$$
$$\left. \left[(\Delta A_2 + B_2) \sin\left(\frac{\Delta x}{\lambda_t}\right) \pm (A_2 - \Delta B_2) \cos\left(\frac{\Delta x}{\lambda_t}\right)\right] e^{-x/\lambda_t} \right\}$$

Nonmagnetic layers

In nonmagnet $J = 0$ and $\beta = 0$
$D_\uparrow = D_\downarrow = D_\perp = D_0$

Particle and spin accumulation

$$\mu_0 = Cx + G$$
$$\boldsymbol{\mu} = \boldsymbol{A}\, e^{x/l_{\mathrm{sf}}} + \boldsymbol{B}\, e^{-x/l_{\mathrm{sf}}}$$

Particle and spin current

$$\frac{1}{\rho(\epsilon_{\mathrm{F}})} j_0 = -2CD_0$$
$$\frac{1}{\rho(\epsilon_{\mathrm{F}})} \boldsymbol{j} = -\frac{2D_0}{l_{\mathrm{sf}}} \left[\boldsymbol{A}e^{x/l_{\mathrm{sf}}} - \boldsymbol{B}e^{-x/l_{\mathrm{sf}}}\right]$$

Boundary conditions and spin transfer torque

Longitudinal spin components

$$e^2 j_0 = (G_\uparrow + G_\downarrow)(\mu_0^{\mathrm{F}} - \mu_0^{\mathrm{N}}) + (G_\uparrow - G_\downarrow)(\mu_z^{\mathrm{F}} - \mu_z^{\mathrm{N}})$$
$$e^2 j_z = (G_\uparrow - G_\downarrow)(\mu_0^{\mathrm{F}} - \mu_0^{\mathrm{N}}) + (G_\uparrow + G_\downarrow)(\mu_z^{\mathrm{F}} - \mu_z^{\mathrm{N}})$$

Transverse spin components

$$e^2 j_x = 2\,\mathrm{Re}\{G_{\uparrow\downarrow}\}(\mu_x^{\mathrm{F}} - \mu_x^{\mathrm{N}}) + 2\,\mathrm{Im}\{G_{\uparrow\downarrow}\}(\mu_y^{\mathrm{F}} - \mu_y^{\mathrm{N}})$$
$$e^2 j_y = 2\,\mathrm{Re}\{G_{\uparrow\downarrow}\}(\mu_y^{\mathrm{F}} - \mu_y^{\mathrm{N}}) - 2\,\mathrm{Im}\{G_{\uparrow\downarrow}\}(\mu_x^{\mathrm{F}} - \mu_x^{\mathrm{N}})$$

Spin current components are continuous acrros the interface

Boundary conditions and spin transfer torque

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Transverse spin components

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$$e^2 j_y = 2\,\mathrm{Re}\{G_{\uparrow\downarrow}\}(\mu_y^{\mathrm{F}} - \mu_y^{\mathrm{N}}) - 2\,\mathrm{Im}\{G_{\uparrow\downarrow}\}(\mu_x^{\mathrm{F}} - \mu_x^{\mathrm{N}})$$

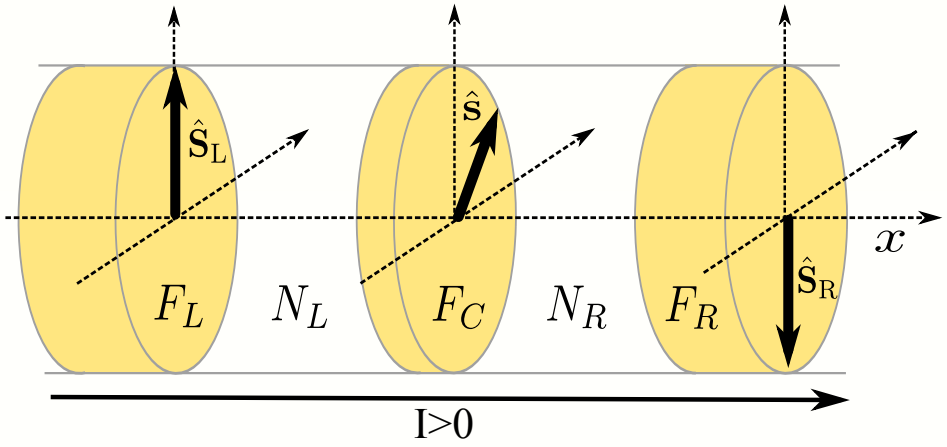
Spin current components are continuous acrros the interface

Spin transfer torque

$$\tau_\parallel = \frac{\hbar}{2}\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \frac{\partial j_y}{\partial x}\,\mathrm{d}x = \frac{\hbar}{2}\left[j_y(x_{\mathrm{R}}) - j_y(x_{\mathrm{L}})\right]$$
$$\tau_\perp = \frac{\hbar}{2}\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} \frac{\partial j_x}{\partial x}\,\mathrm{d}x = \frac{\hbar}{2}\left[j_x(x_{\mathrm{R}}) - j_x(x_{\mathrm{L}})\right]$$

Outline

Current-induced dynamics



Landau-Lifshitz-Gilbert equation

$$\frac{\mathrm{d}\hat{\boldsymbol{s}}}{\mathrm{d}t} + \alpha\hat{\boldsymbol{s}} \times \frac{\mathrm{d}\hat{\boldsymbol{s}}}{\mathrm{d}t} = \boldsymbol{\Gamma} \qquad \text{where} \qquad \boldsymbol{\Gamma} = -|\gamma_\mathrm{g}|\mu_0\, \hat{\boldsymbol{s}} \times \mathbf{H}_\mathrm{eff} + \frac{\gamma_\mathrm{g}}{M_\mathrm{s} d}\boldsymbol{\tau}$$

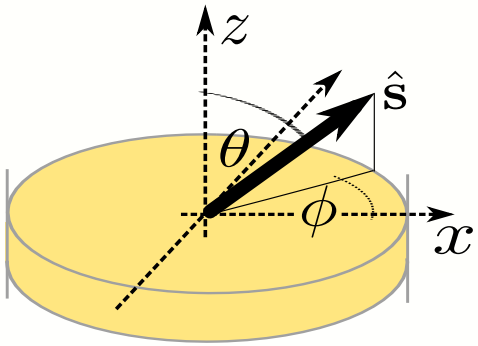
where $\boldsymbol{\tau} = \boldsymbol{\tau}_{\parallel} + \boldsymbol{\tau}_{\perp}$ is the spin transfer torque

Spin transfer torque

$$\boldsymbol{\tau}_{\parallel} = I\,\hat{\boldsymbol{s}} \times \left[\hat{\boldsymbol{s}} \times \left(a_\mathrm{L}\hat{\boldsymbol{S}}_\mathrm{L} + a_\mathrm{R}\hat{\boldsymbol{S}}_\mathrm{R}\right)\right]$$

$$\boldsymbol{\tau}_{\perp} = I\,\hat{\boldsymbol{s}} \times \left(b_\mathrm{L}\hat{\boldsymbol{S}}_\mathrm{L} + b_\mathrm{R}\hat{\boldsymbol{S}}_\mathrm{R}\right)$$

Local spherical coordinates



Local coordinates

$$\hat{\mathbf{e}}_\phi = (\hat{\mathbf{e}}_z \times \hat{\boldsymbol{s}})/\sin\theta,\ \hat{\mathbf{e}}_\theta = \hat{\mathbf{e}}_\phi \times \hat{\boldsymbol{s}}$$
$$\hat{\boldsymbol{s}} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$$

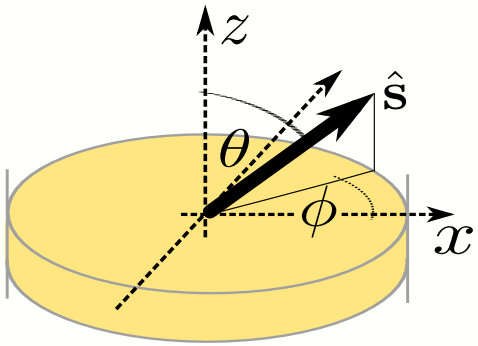
Landau-Lifshitz-Gilbert equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\theta\\ \phi\end{pmatrix} = \frac{1}{1+\alpha^2}\begin{pmatrix}1 & \alpha\\ -\alpha/\sin\theta & 1/\sin\theta\end{pmatrix}\begin{pmatrix}\omega_\theta\\ \omega_\phi\end{pmatrix}$$

Frequencies

$$\omega_\theta = \Gamma\cdot\hat{\mathbf{e}}_\theta \text{ and } \omega_\phi = \Gamma\cdot\hat{\mathbf{e}}_\phi$$
$$\omega_\theta = \omega_\theta^h + \omega_\theta^\tau$$
$$\omega_\phi = \omega_\phi^h + \omega_\phi^\tau$$

Local spherical coordinates



Local coordinates

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Landau-Lifshitz-Gilbert equation

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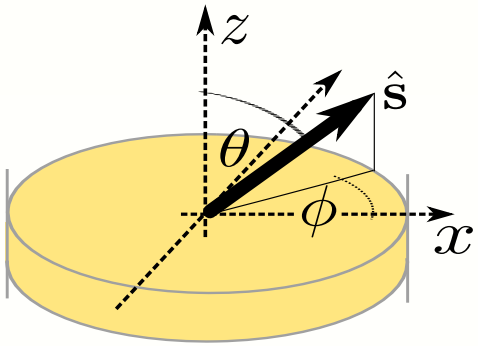
Frequencies

$$\omega_\theta = \Gamma\cdot\hat{\mathbf{e}}_\theta \text{ and } \omega_\phi = \Gamma\cdot\hat{\mathbf{e}}_\phi$$
$$\omega_\theta = \omega_\theta^h + \omega_\theta^\tau$$
$$\omega_\phi = \omega_\phi^h + \omega_\phi^\tau$$

$$\omega_\theta^h = |\gamma_\mathrm{g}|\mu_0\left[H_{||}\cos\phi\sin\phi\sin\theta - H_\mathrm{app}\sin(\Theta-\phi)\right]$$
$$\omega_\phi^h = |\gamma_\mathrm{g}|\mu_0\left[\left(H_\mathrm{dz} - H_\mathrm{dy} + H_{||}\cos(2\phi)\right)\sin\theta - H_\mathrm{app}\cos(\Theta-\phi)\right]\cos\theta$$

Local spherical coordinates



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Landau-Lifshitz-Gilbert equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\theta\\ \phi\end{pmatrix} = \frac{1}{1+\alpha^2}\begin{pmatrix}1 & \alpha\\ -\alpha/\sin\theta & 1/\sin\theta\end{pmatrix}\begin{pmatrix}\omega_\theta\\ \omega_\phi\end{pmatrix}$$

Frequencies

$$\omega_\theta = \Gamma\cdot\hat{\mathbf{e}}_\theta \text{ and } \omega_\phi = \Gamma\cdot\hat{\mathbf{e}}_\phi$$
$$\omega_\theta = \omega_\theta^h + \omega_\theta^\tau$$
$$\omega_\phi = \omega_\phi^h + \omega_\phi^\tau$$

$$\omega_\theta^\tau = \frac{|\gamma_\mathrm{g}|I}{dM_\mathrm{s}}(a\ \cos\theta\cos\phi - b\ \sin\phi)$$
$$\omega_\theta^\tau = -\frac{|\gamma_\mathrm{g}|I}{dM_\mathrm{s}}(a\ \sin\phi + b\ \cos\theta\cos\phi)$$

Linear response

Close to equilibrium: $\theta = \theta_0 + \delta\theta$, $\phi = \phi_0 + \delta\phi$

$$\omega_\theta(\hat{\mathbf{s}}) = \omega_\theta(\hat{\mathbf{s}}_0) + \frac{\partial \omega_\theta}{\partial \theta}|_{\hat{\mathbf{s}}_0}\ \delta\theta + \frac{\partial \omega_\theta}{\partial \phi}|_{\hat{\mathbf{s}}_0}\ \delta\phi$$

$$\omega_\phi(\hat{\mathbf{s}}) = \omega_\phi(\hat{\mathbf{s}}_0) + \frac{\partial \omega_\phi}{\partial \theta}|_{\hat{\mathbf{s}}_0}\ \delta\theta + \frac{\partial \omega_\phi}{\partial \phi}|_{\hat{\mathbf{s}}_0}\ \delta\phi$$

Considering $\omega_\theta^\tau(\hat{\mathbf{s}}) \simeq \omega_\theta^\tau(\hat{\mathbf{s}}_0)$, and $\omega_\phi^\tau(\hat{\mathbf{s}}) \simeq \omega_\phi^\tau(\hat{\mathbf{s}}_0)$
$\dot{\theta}_0 = \dot{\phi}_0 = 0$, $\omega_\theta^h(\hat{\mathbf{s}}_0) = \omega_\phi^h(\hat{\mathbf{s}}_0) = 0$
for $\theta_0 = \pi/2$

Linear response

Close to equilibrium: $\theta = \theta_0 + \delta\theta$, $\phi = \phi_0 + \delta\phi$

$$\omega_\theta(\hat{\mathbf{s}}) = \omega_\theta(\hat{\mathbf{s}}_0) + \frac{\partial \omega_\theta}{\partial \theta}|_{\hat{\mathbf{s}}_0}\,\delta\theta + \frac{\partial \omega_\theta}{\partial \phi}|_{\hat{\mathbf{s}}_0}\,\delta\phi$$

$$\omega_\phi(\hat{\mathbf{s}}) = \omega_\phi(\hat{\mathbf{s}}_0) + \frac{\partial \omega_\phi}{\partial \theta}|_{\hat{\mathbf{s}}_0}\,\delta\theta + \frac{\partial \omega_\phi}{\partial \phi}|_{\hat{\mathbf{s}}_0}\,\delta\phi$$

$$a_0(\phi_0) = \frac{|\gamma_\mathrm{g}|}{dM_\mathrm{s}} a(\phi_0)\sin\phi_0$$

$$b_0(\phi_0) = \frac{|\gamma_\mathrm{g}|}{dM_\mathrm{s}} b(\phi_0)\sin\phi_0$$

Considering $\omega_\theta^\tau(\hat{\mathbf{s}}) \simeq \omega_\theta^\tau(\hat{\mathbf{s}}_0)$, and $\omega_\phi^\tau(\hat{\mathbf{s}}) \simeq \omega_\phi^\tau(\hat{\mathbf{s}}_0)$
$\dot{\theta}_0 = \dot{\phi}_0 = 0$, $\omega_\theta^h(\hat{\mathbf{s}}_0) = \omega_\phi^h(\hat{\mathbf{s}}_0) = 0$
for $\theta_0 = \pi/2$

Solution

For $I = I_0 e^{i\omega t}$ we consider
$\delta\theta = A_\theta e^{i\omega t}$, and $\delta\phi = A_\phi e^{i\omega t}$

Linearized Landau-Lifshitz-Gilbert equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\delta\theta\\ \delta\phi\end{pmatrix} = \begin{pmatrix}\alpha\omega_1 & -\omega_2\\ \omega_1 & \alpha\omega_2\end{pmatrix}\begin{pmatrix}\delta\theta\\ \delta\phi\end{pmatrix} + \begin{pmatrix}b_0 + \alpha a_0\\ a_0 - \alpha b_0\end{pmatrix} I$$

where $I = -|e|\, j_0$ is the current density

$$\omega_1(\phi_0,\Theta) = -|\gamma_\mathrm{g}|\mu_0\left[H_{\mathrm{d}x} - H_{\mathrm{d}y} + H_{||}\cos^2\Theta - H_\mathrm{int}\cos\phi_0 + H_\mathrm{app}\cos(\Theta-\phi_0)\right]$$
$$\omega_2(\phi_0,\Theta) = -|\gamma_\mathrm{g}|\mu_0\left[H_{||}\cos(2\phi_0) - H_\mathrm{int}\cos\phi_0 + H_\mathrm{app}\cos(\Theta-\phi_0)\right]$$

Second harmonic voltage

Magnetic susceptibility to spin transfer torque

$$\chi_\phi(\phi_0, \Theta, \omega) = \frac{A_\phi}{I_0} = \frac{|\gamma_g|}{M_s d} \frac{b(\phi_0)\omega_1 + i\,[a(\phi_0) - \alpha b(\phi_0)]\,\omega}{(\omega_1\omega_2 - \omega^2) - i\,\alpha(\omega_1 + \omega_2)\,\omega} \sin\phi_0$$

$\omega_0 = \omega_1\omega_2$ is the resonance frequency

For small current frequency ($\omega \to 0$)

$$\chi_\phi(\phi_0, \Theta) = \frac{|\gamma_g|}{M_s d} \frac{b(\phi_0)}{\omega_2(\phi_0, \Theta)} \sin\phi_0$$

Second harmonic voltage

$$U = R(t)I(t)$$

The angle between $\hat{s} = (\theta, \phi)$ and $\hat{S}$ is

$$\cos\phi' = \hat{s}\cdot\hat{S} = \cos\phi\sin\theta = \nu(\theta,\phi)$$

Close to $\hat{s}_0 = (\phi_0, \theta_0)$

$$R(\nu) = R(\nu_0) + \frac{\partial R(\nu)}{\partial \nu}\frac{\partial \nu}{\partial \theta}\,|_{\hat{s}_0}\,\delta\theta + \frac{\partial R(\nu)}{\partial \nu}\frac{\partial \nu}{\partial \phi}\,|_{\hat{s}_0}\,\delta\phi$$

where $\nu_0 = \nu(\theta_0,\phi_0) = \cos\phi_0\sin\theta_0$

$$R = R(\phi_0) - \frac{\partial R}{\partial \nu}\,|_{\hat{s}_0}\,\sin\phi_0\,\delta\phi$$

where $\delta\phi = \mathrm{Re}\left\{A_\phi e^{i\omega t}\right\}$

$$R(t) = R(\phi_0) - \frac{\partial R}{\partial \nu}\,|_{\phi_0}\,\sin\phi_0\, I_0\,\mathrm{Re}\left\{\chi_\phi e^{i\omega t}\right\}$$

$$U = I_0 R(\phi_0)\cos(\omega t) - \frac{I_0^2}{4}\frac{\partial R}{\partial \nu}\,|_{\phi_0}\,\sin\phi_0 \times \left(\chi_\phi e^{i\omega t} + \chi_\phi^* e^{-i\omega t}\right)\left(e^{i\omega t} + e^{-i\omega t}\right)$$

DC voltage

$$U_0 = -\frac{I_0^2}{2}\frac{\partial R}{\partial \nu}\,|_{\phi_0}\,\sin\phi_0\,\chi'_\phi(\phi_0,\Theta,\omega)$$

Second harmonic AC voltage

$$U_{2\omega}(t) = -\frac{I_0^2}{2}\frac{\partial R}{\partial \nu}\,|_{\phi_0}\,\sin\phi_0\,\mathrm{Re}\left\{\chi_\phi e^{i(2\omega)t}\right\} = -\frac{I_0^2}{2}\frac{\partial R}{\partial \nu}\,|_{\phi_0}\,\sin\phi_0 \times \left[\chi'_\phi\cos(2\omega t) - \chi''_\phi\sin(2\omega t)\right]$$

where $\chi_\phi = \chi'_\phi + i\,\chi''_\phi$

Lock-in amplifier measurement

Lock-in measurement

$$U_{\text{out}} = \frac{1}{T}\int_0^T \sin(\omega' t + \psi) U_{\text{in}}(t)\mathrm{d}t$$

To obtain phase-independent signal we use
$U_{\text{ref},1} = \sin(\omega' t + \psi)$, and $U_{\text{ref},2} = \cos(\omega' t + \psi)$

$$U_{\text{out},1} \propto \frac{1}{T}\int_0^T \mathrm{d}t\ \sin(2\omega t + \psi)\left[\chi'_\phi \cos(2\omega t) - \chi''_\phi \sin(2\omega t)\right]$$

$$U_{\text{out},2} \propto \frac{1}{T}\int_0^T \mathrm{d}t\ \cos(2\omega t + \psi)\left[\chi'_\phi \cos(2\omega t) - \chi''_\phi \sin(2\omega t)\right]$$

Lock-in amplifier measurement

Lock-in measurement

$$U_{\text{out}} = \frac{1}{T}\int_0^T \sin(\omega' t + \psi) U_{\text{in}}(t)\mathrm{d}t$$

To obtain phase-independent signal we use
$U_{\text{ref},1} = \sin(\omega' t + \psi)$, and $U_{\text{ref},2} = \cos(\omega' t + \psi)$

$$U_{\text{out},1} \propto \frac{1}{2}\left(\chi'_\phi \sin\psi - \chi''_\phi \cos\psi\right)$$
$$U_{\text{out},2} \propto \frac{1}{2}\left(\chi'_\phi \cos\psi + \chi''_\phi \sin\psi\right)$$

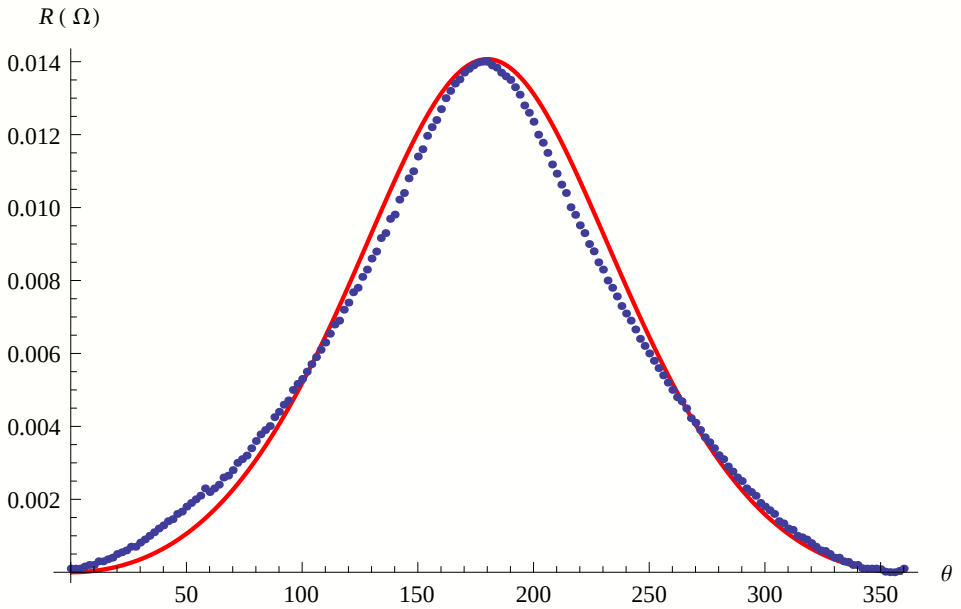
One measures $U_{2\omega} \propto \frac{1}{2}\sqrt{(\chi'_\phi)^2 + (\chi''_\phi)^2} = \frac{1}{2}|\chi_\phi|$ which is a dc voltage

Second harmonic response

$$U_{2\omega} = -\frac{I_0^2}{4}\frac{\partial R}{\partial \nu}\Big|_{\phi_0} \sin\phi_0 \, |\chi_\phi|$$

Outline

Fit to experimental results (EPFL)

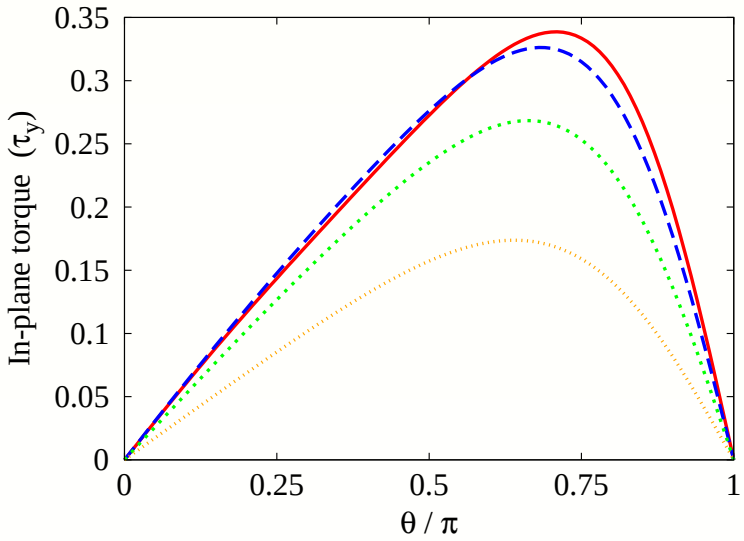


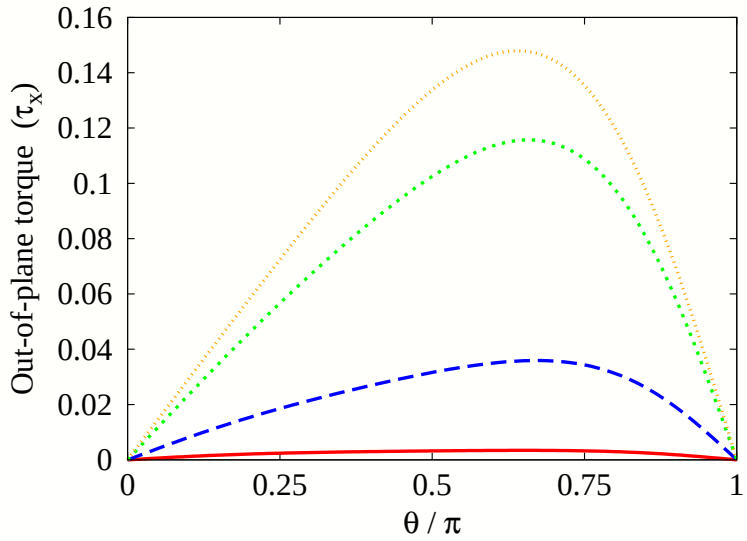
material	ρ^* ($\mu\Omega$cm)	β	λ_{sf} (nm)
Cu	0.5	0	100.0
CoFe	20.0	0.65	12.0

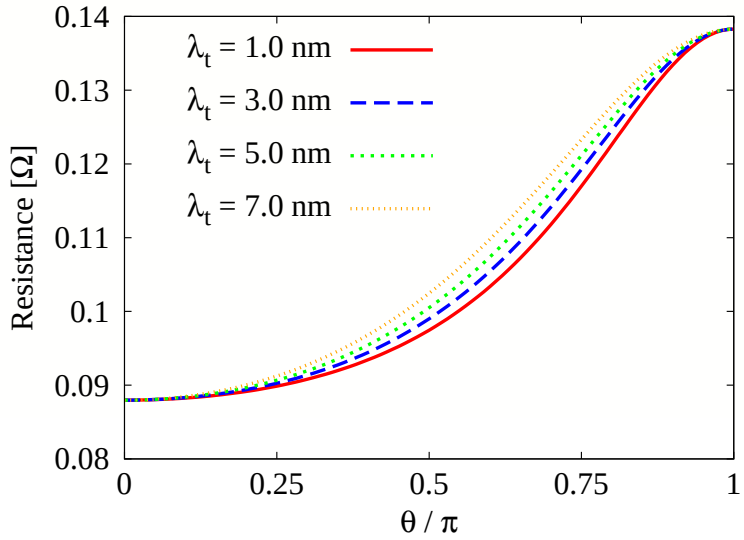
interface	R^* (fΩm^2)	γ	Re $\{G_{\uparrow\downarrow}\}$ $(1/\text{f}\Omega\text{m}^2)$	Im $\{G_{\uparrow\downarrow}\}$ $(1/\text{f}\Omega\text{m}^2)$
CoFe / Cu	0.5	0.55	1.80	0.02

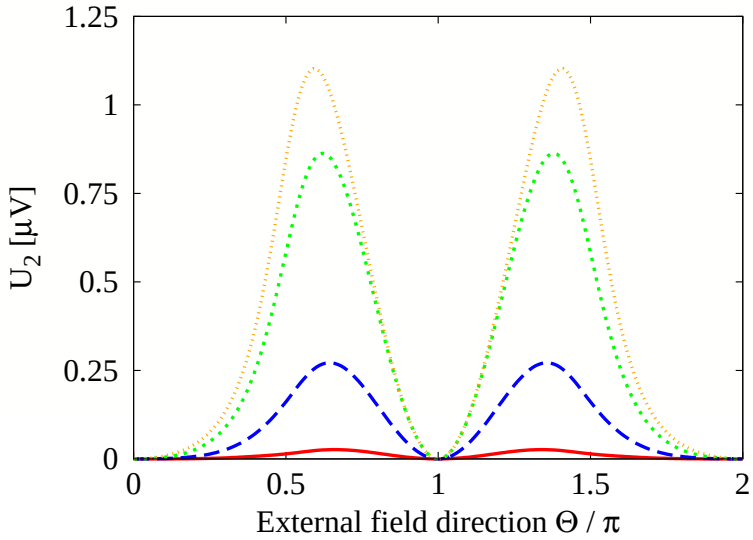
Numerical results: Single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



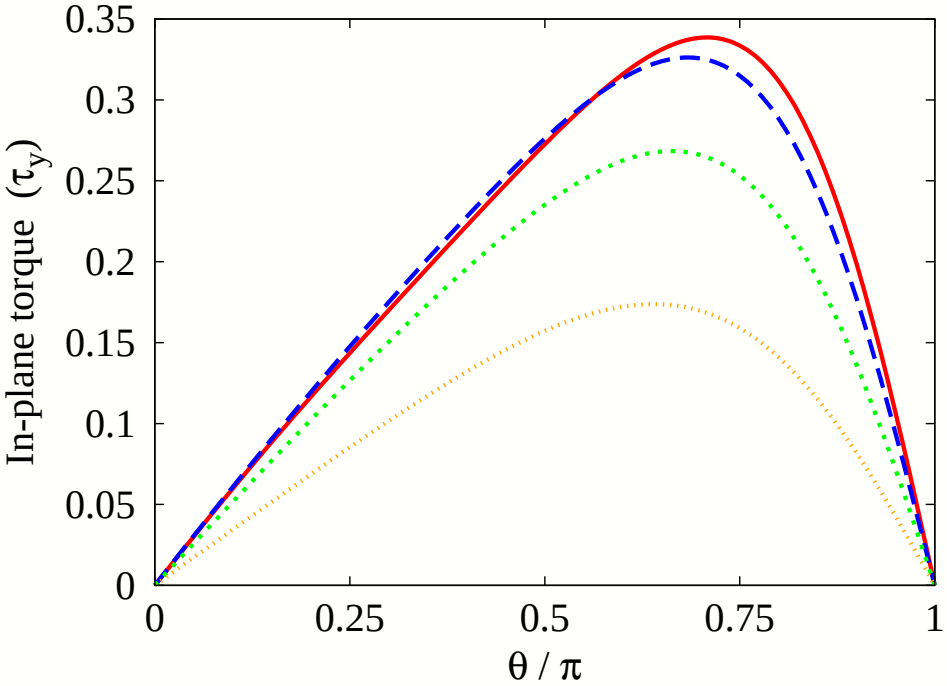


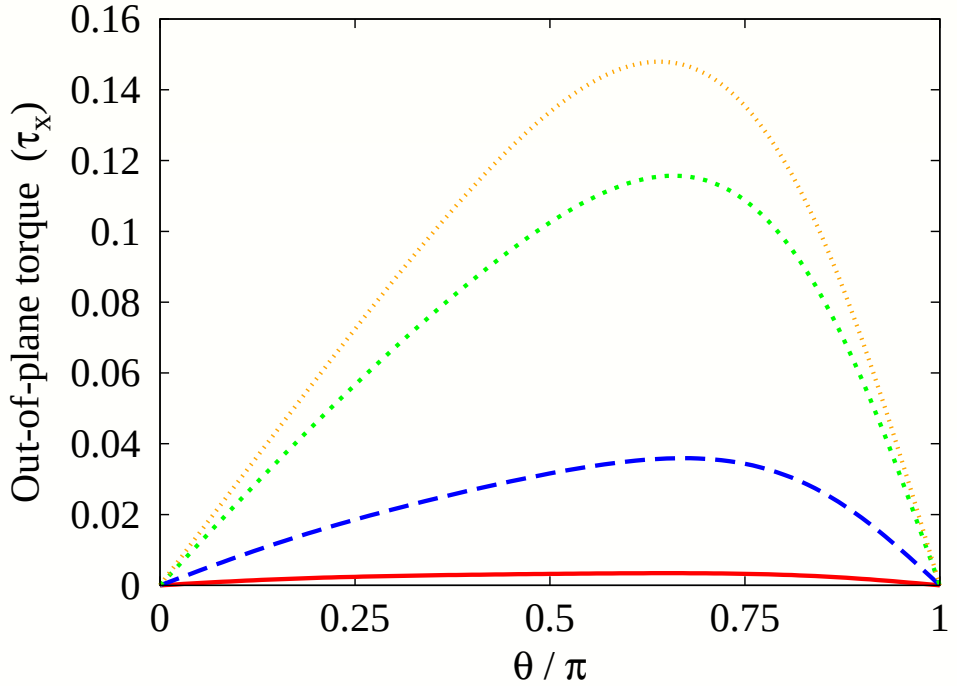


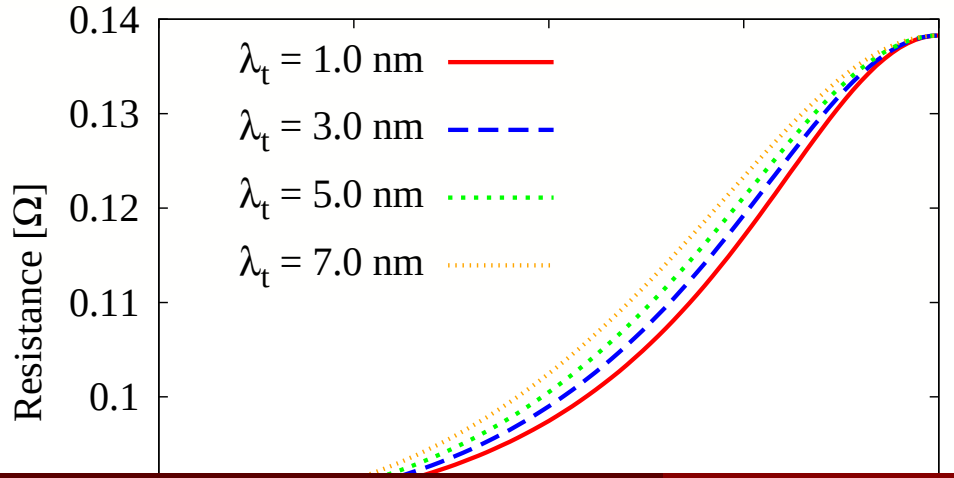


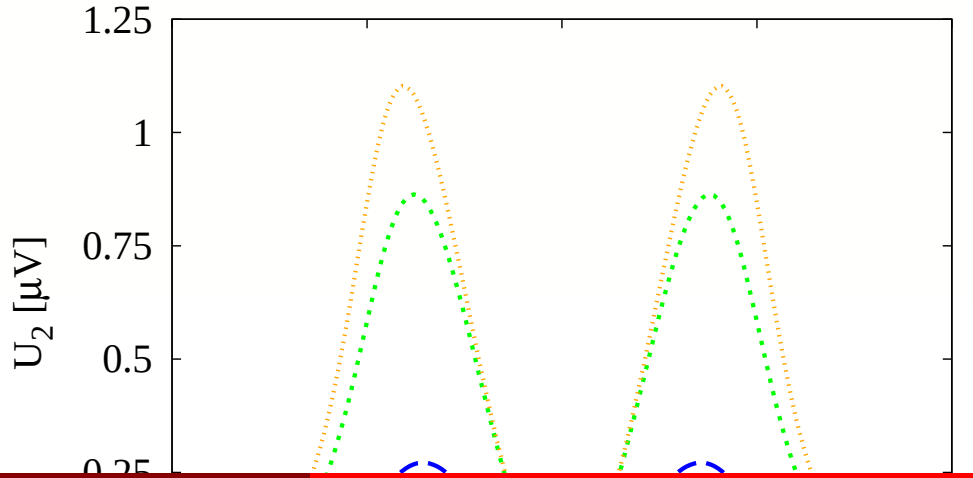
Numerical results: Single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



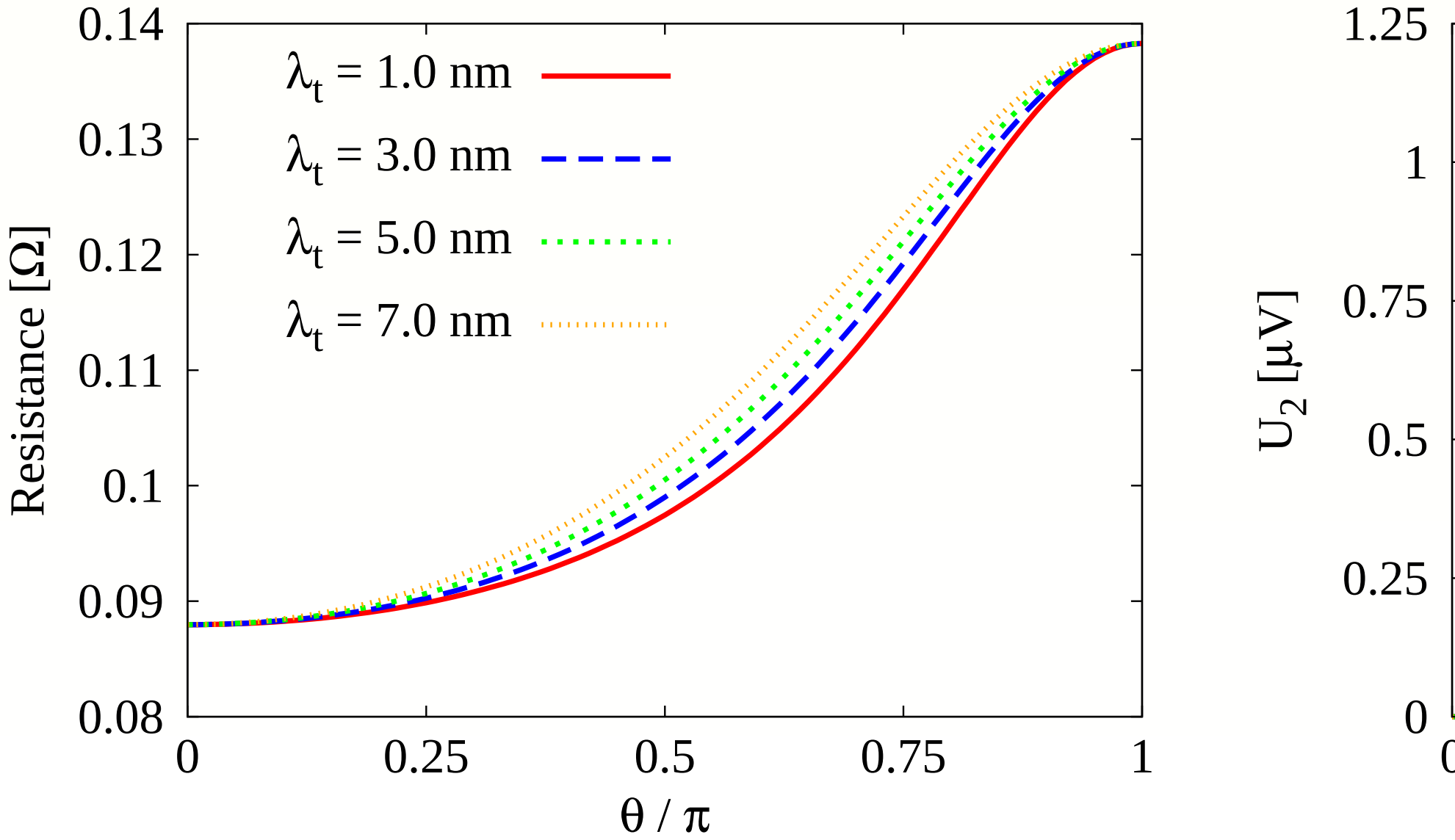






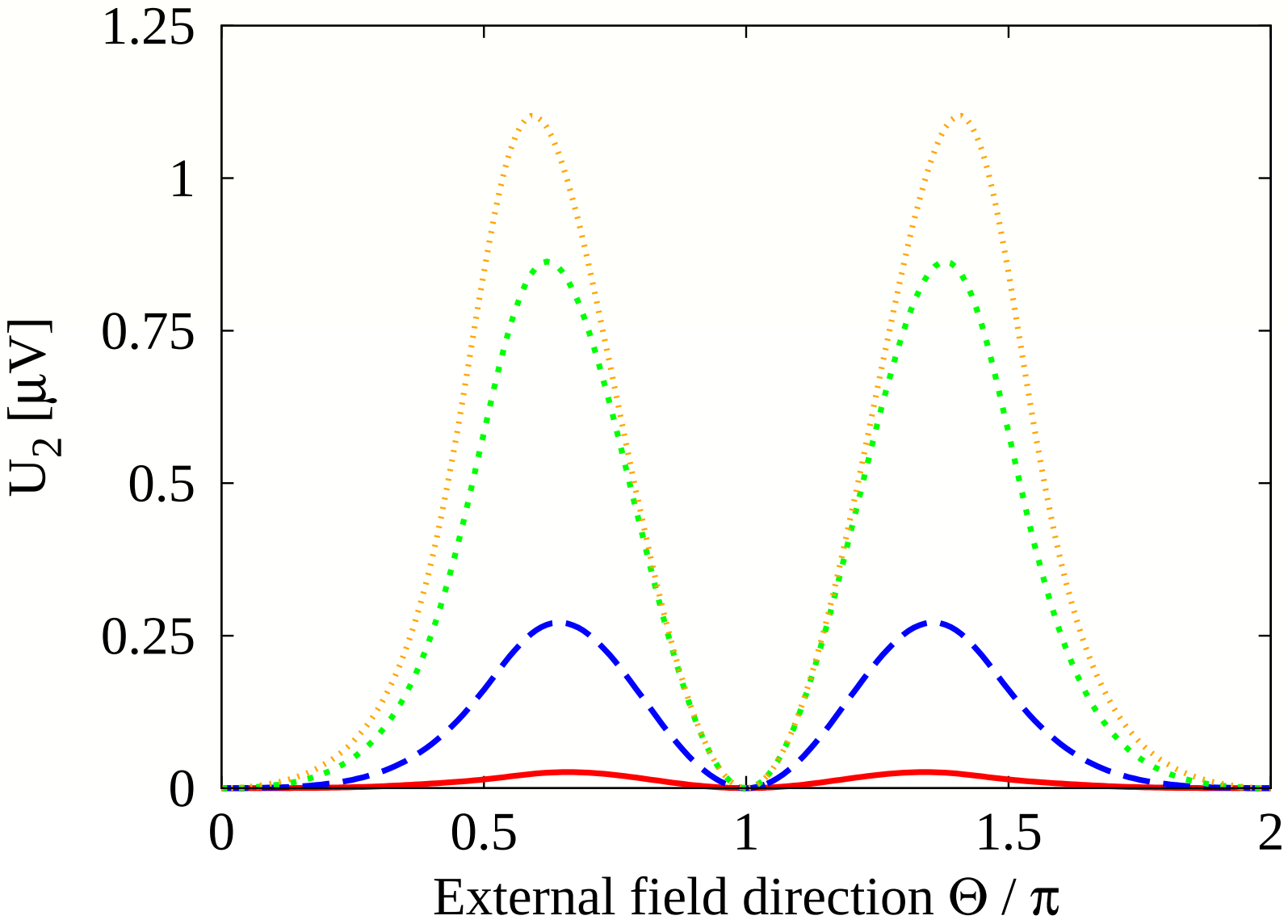
Numerical results: Single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



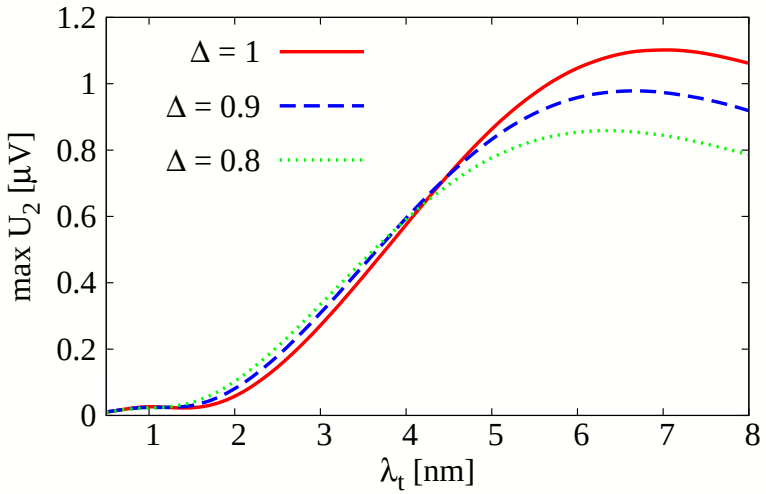
Numerical results: Single spin valve

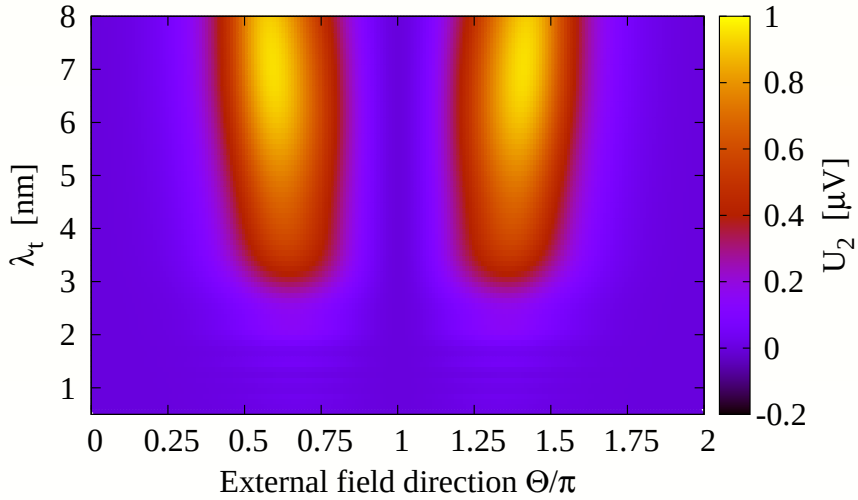
Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu

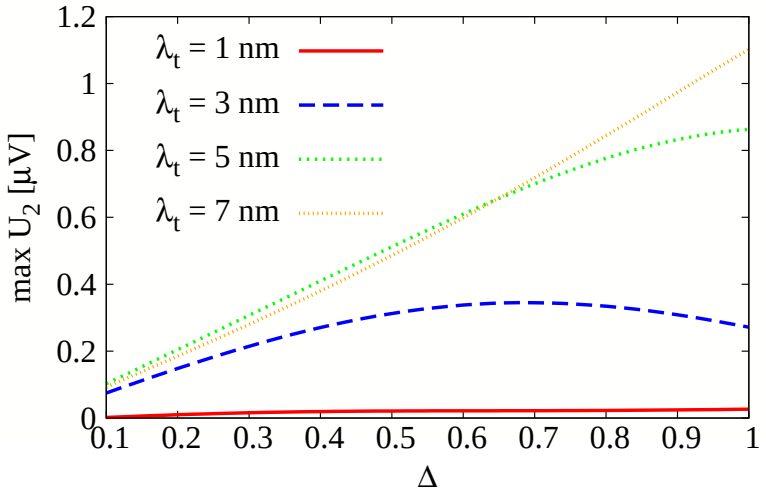


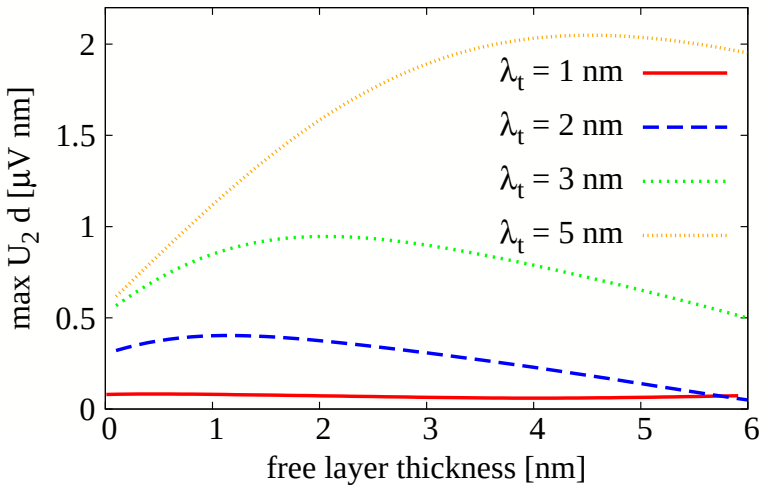
Numerical results: single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



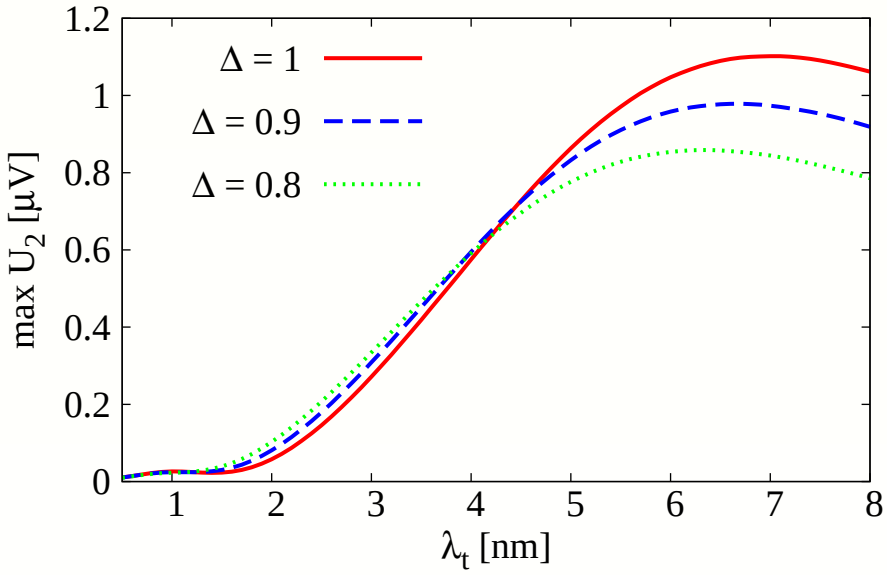


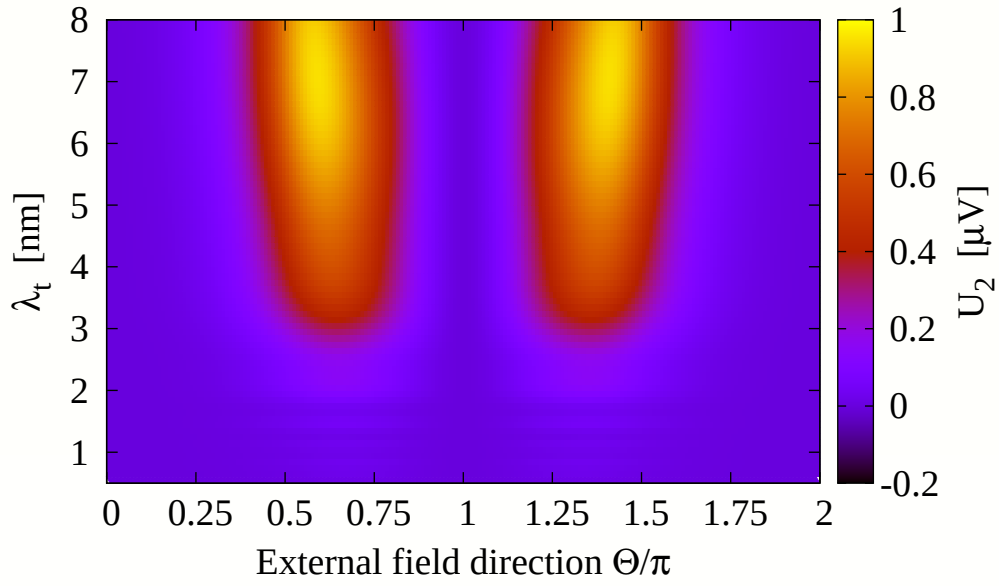


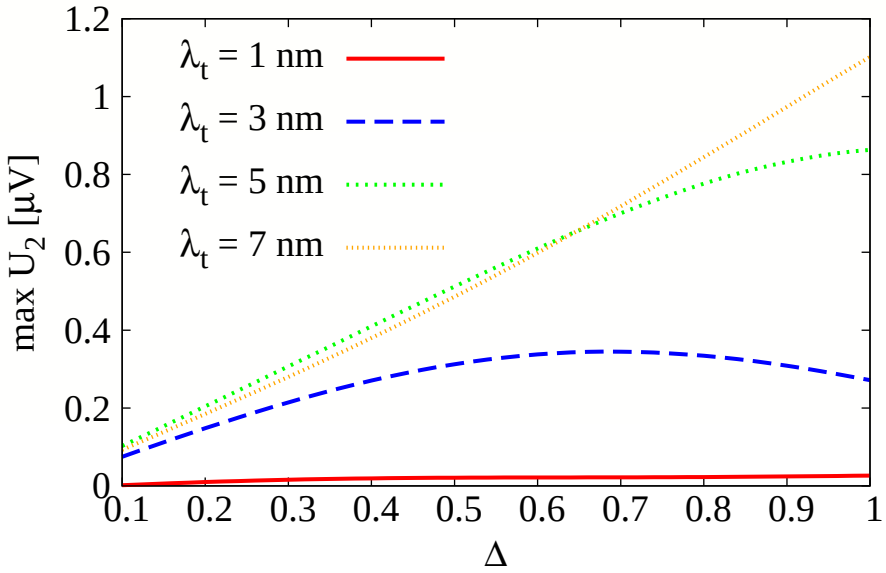


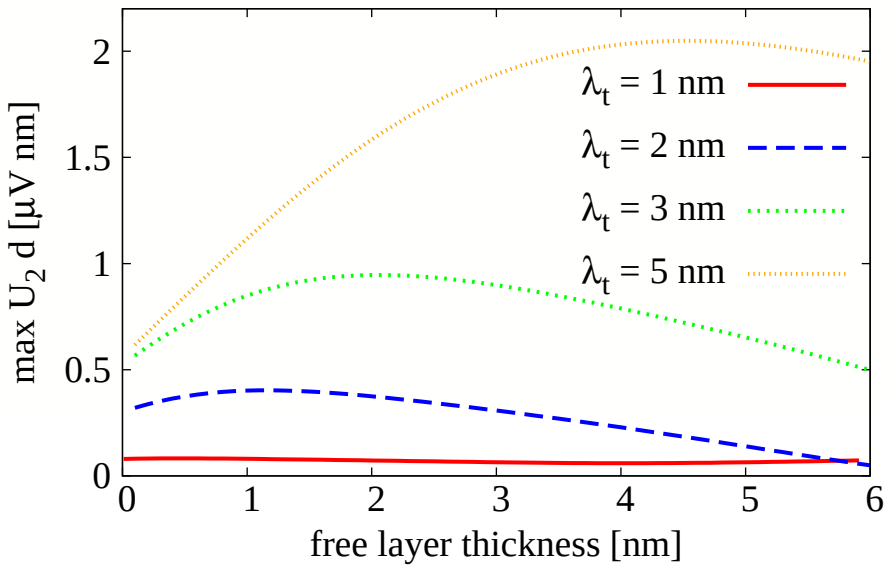
Numerical results: single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



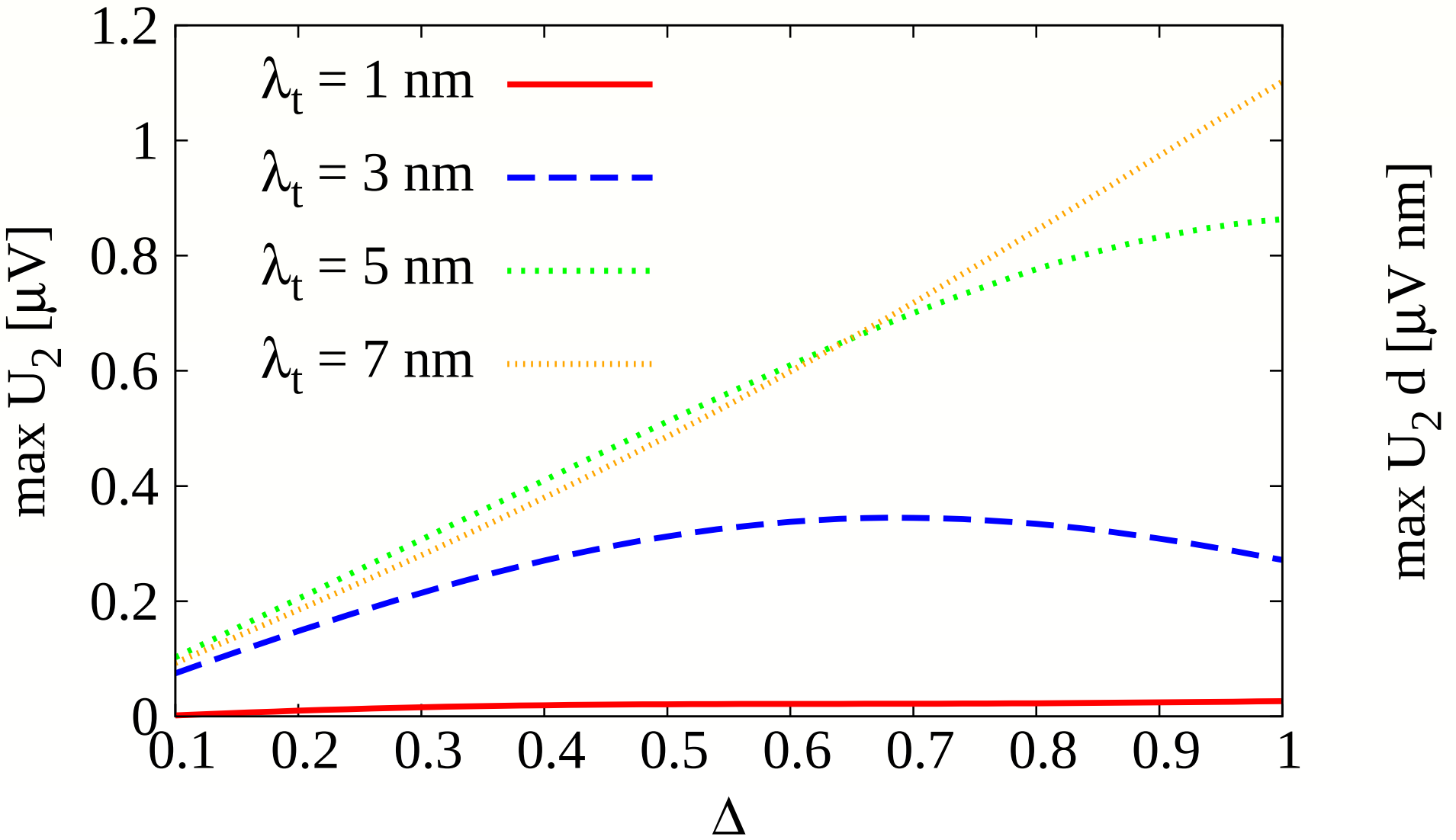






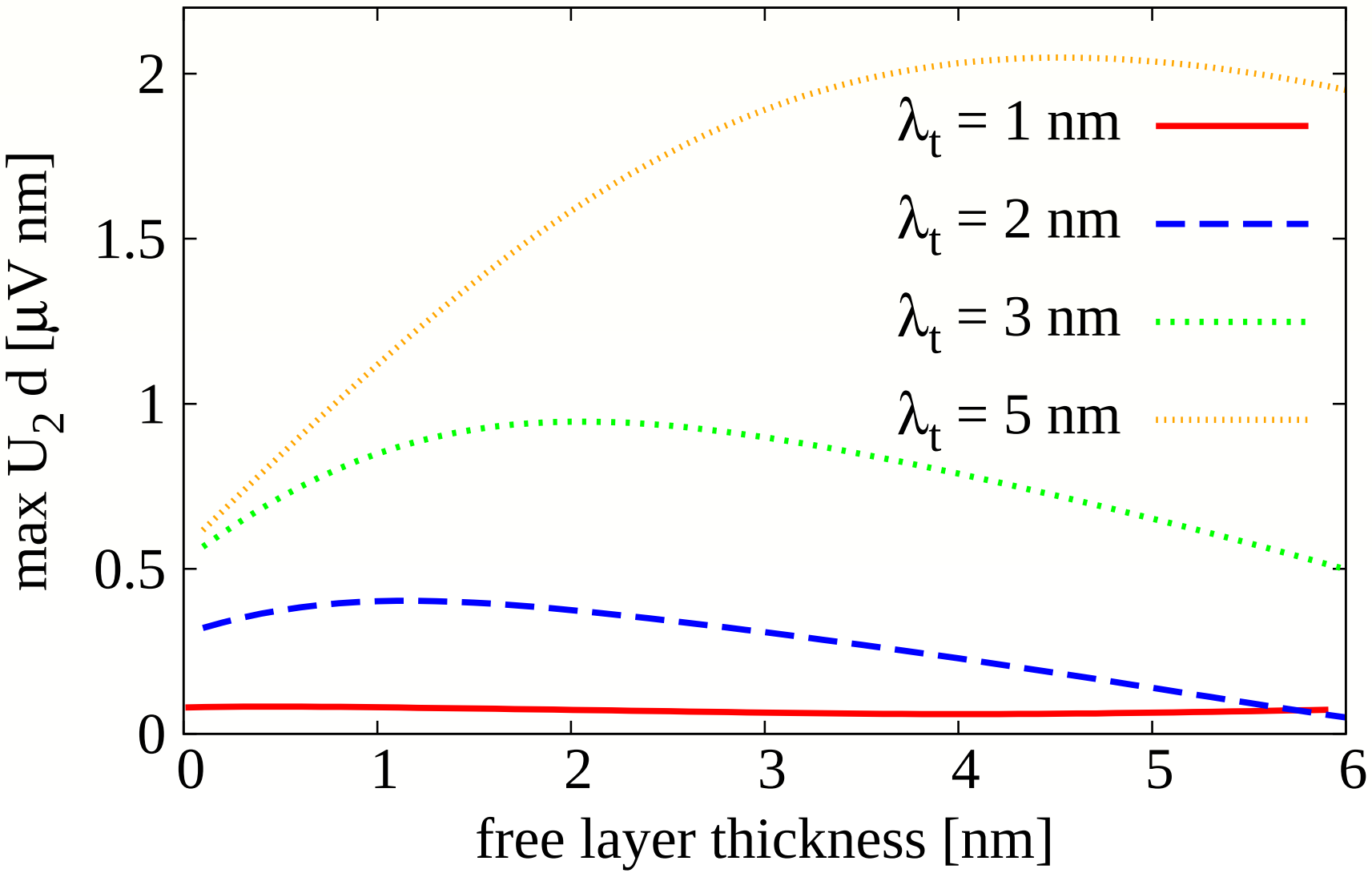
Numerical results: single spin valve

Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu



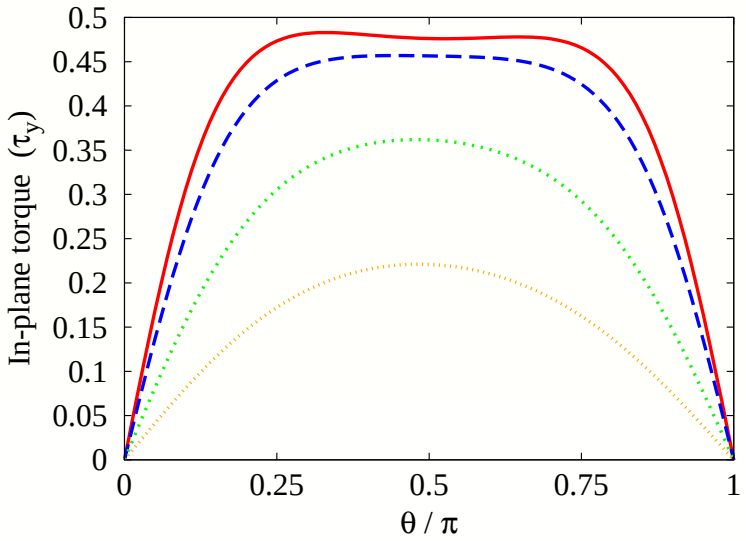
Numerical results: single spin valve

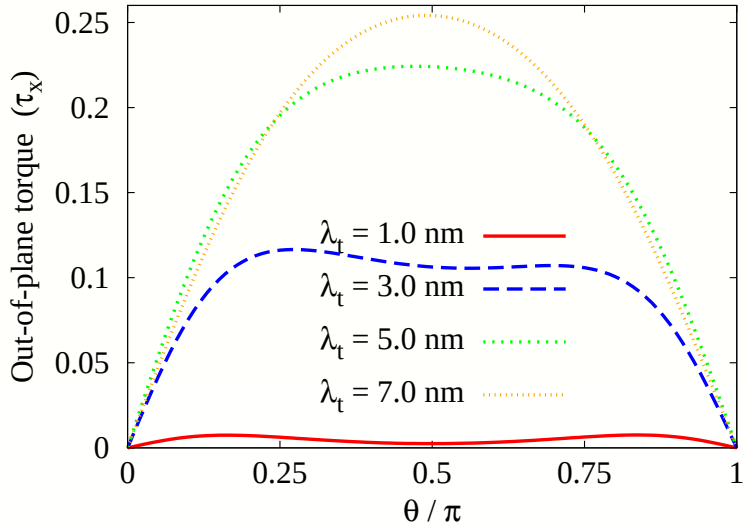
Cu – CoFe(12) / Cu(8) / CoFe(5) – Cu

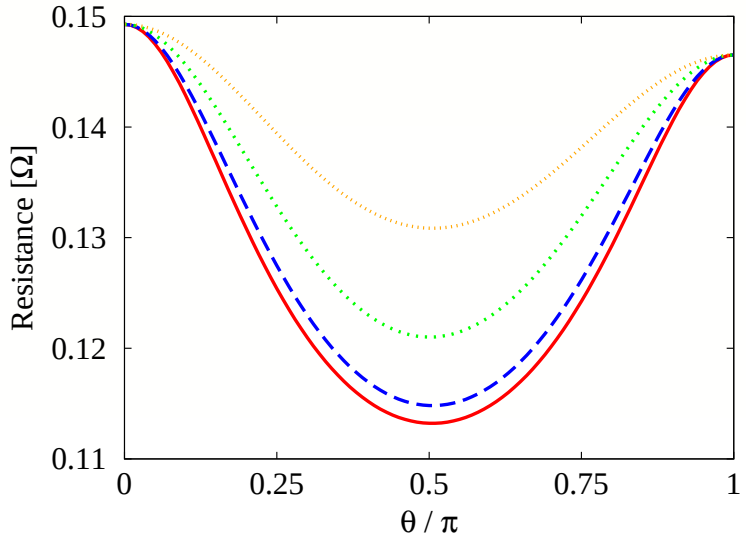


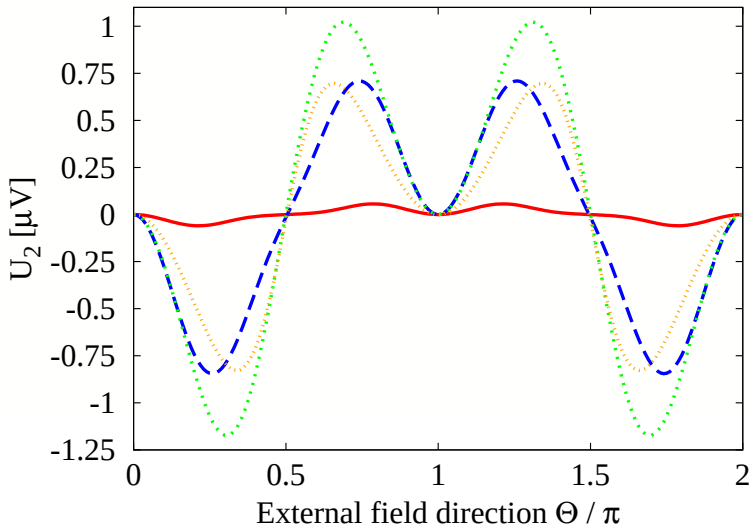
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



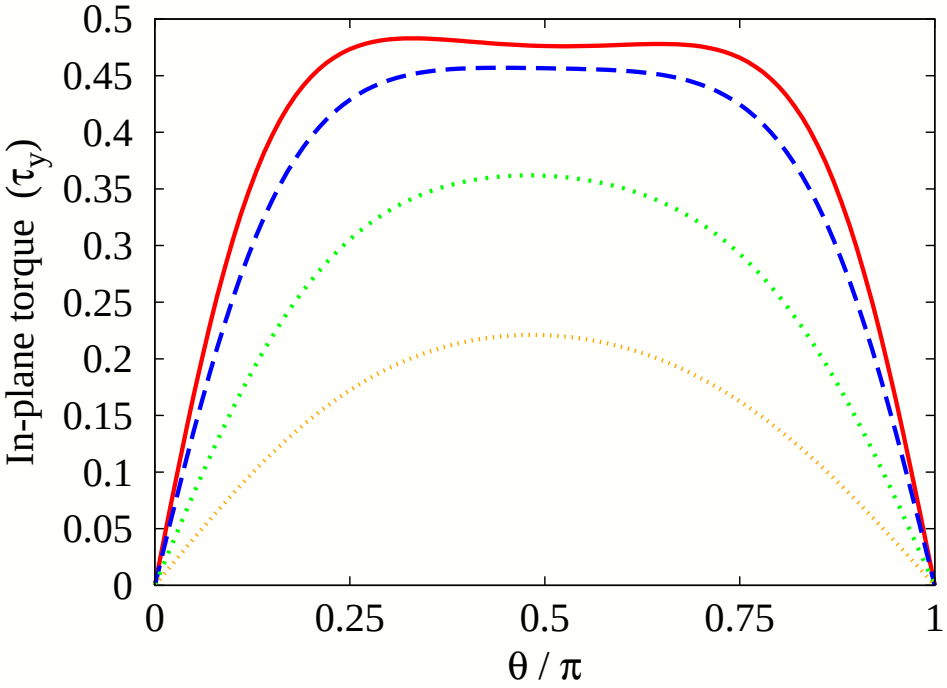


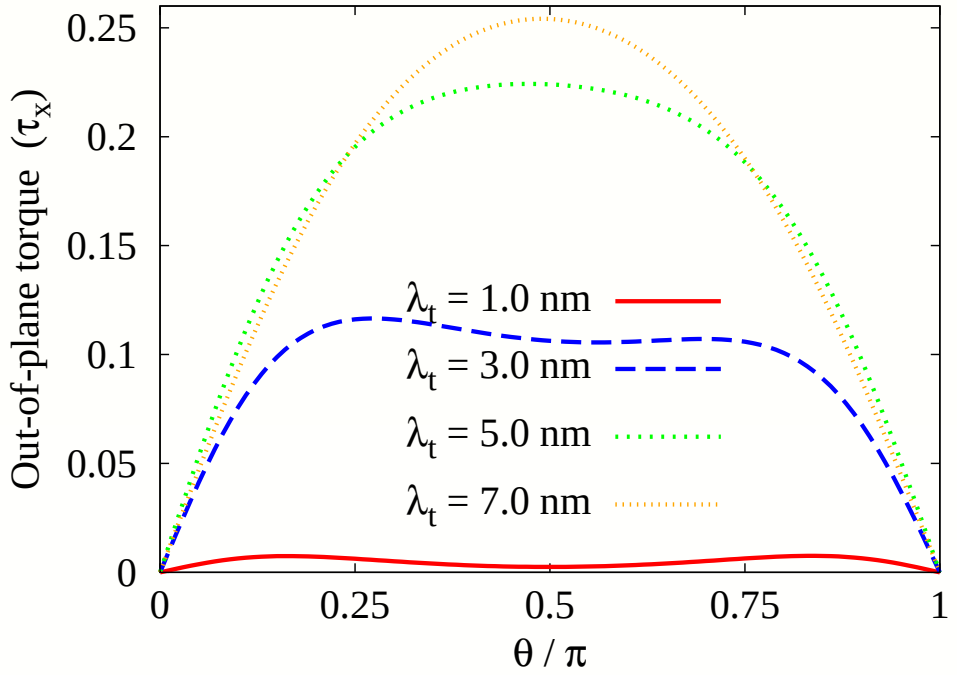


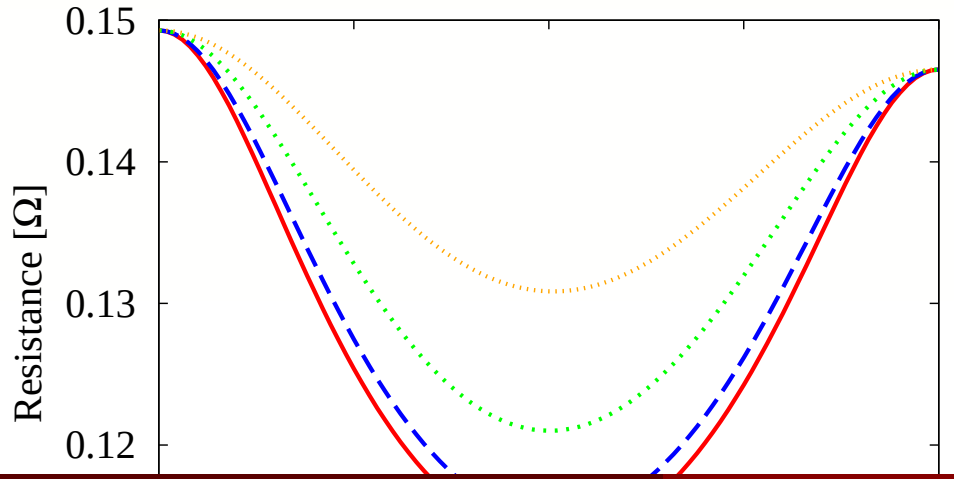


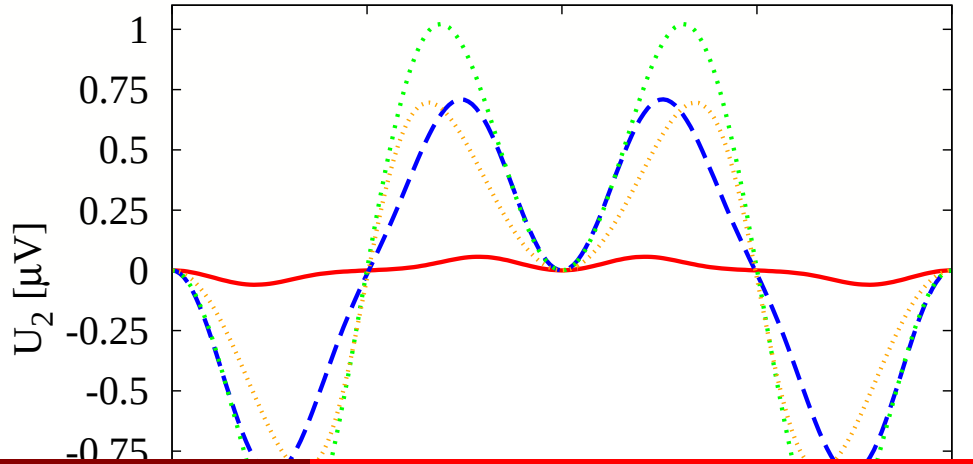
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



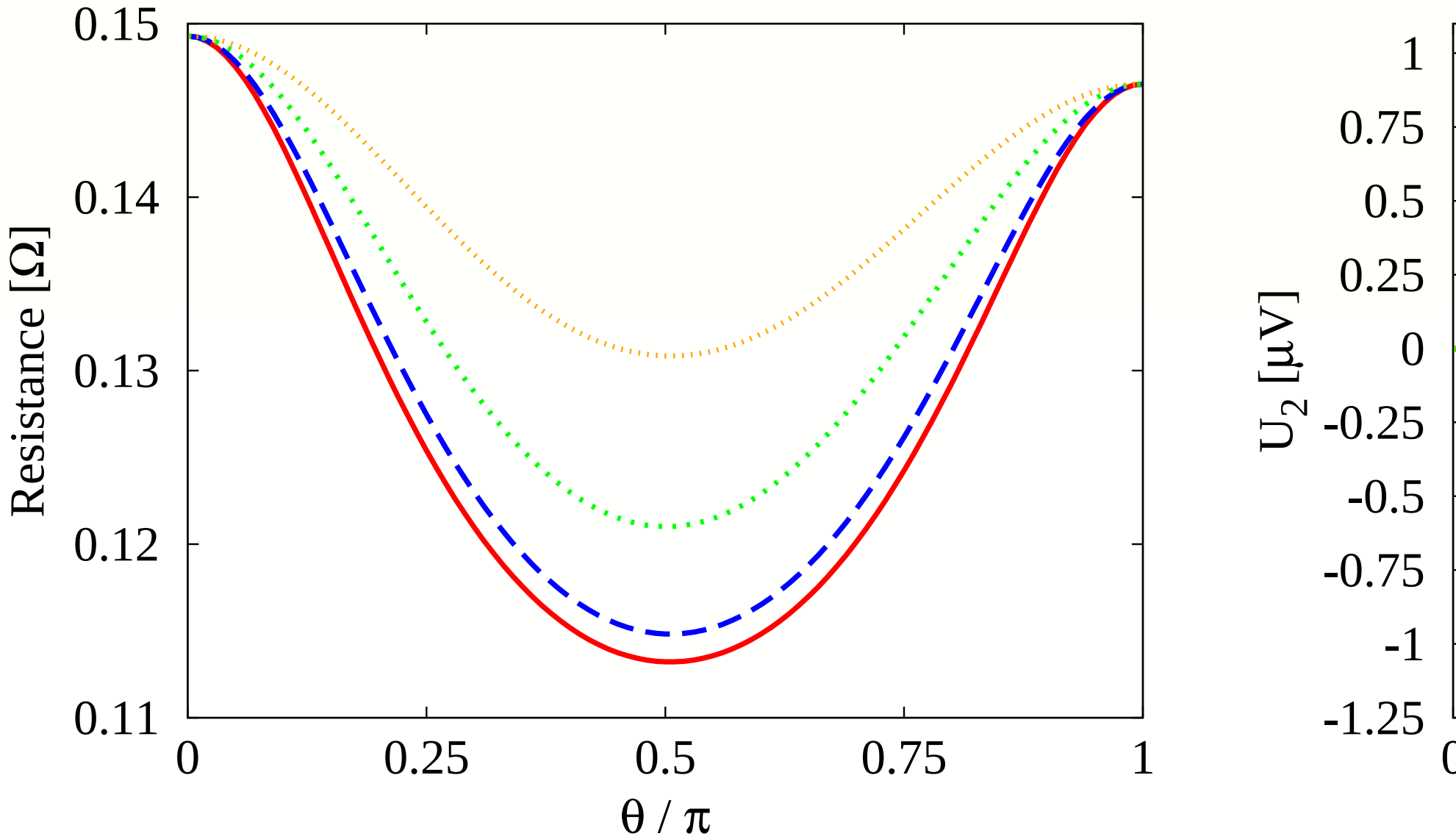






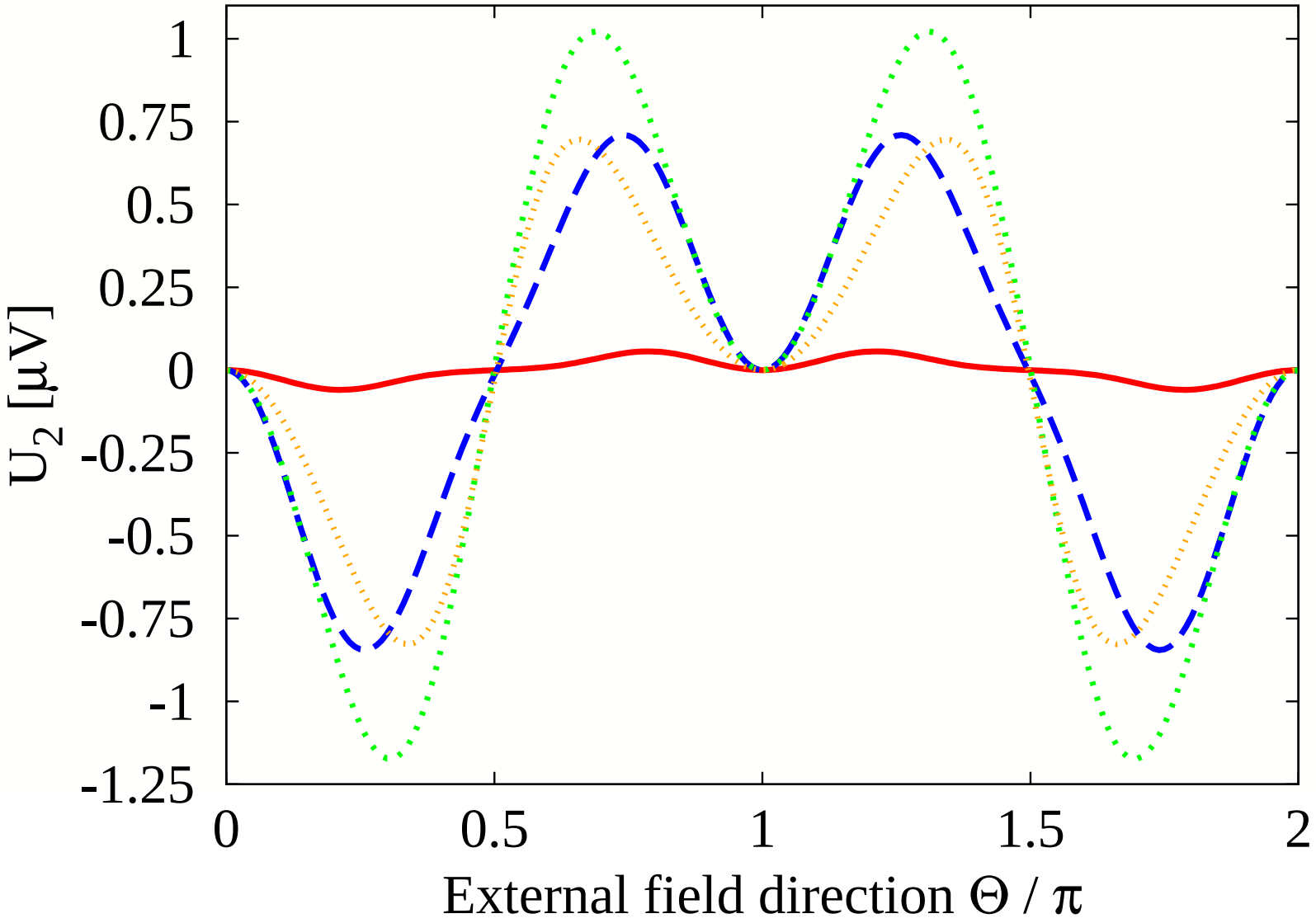
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



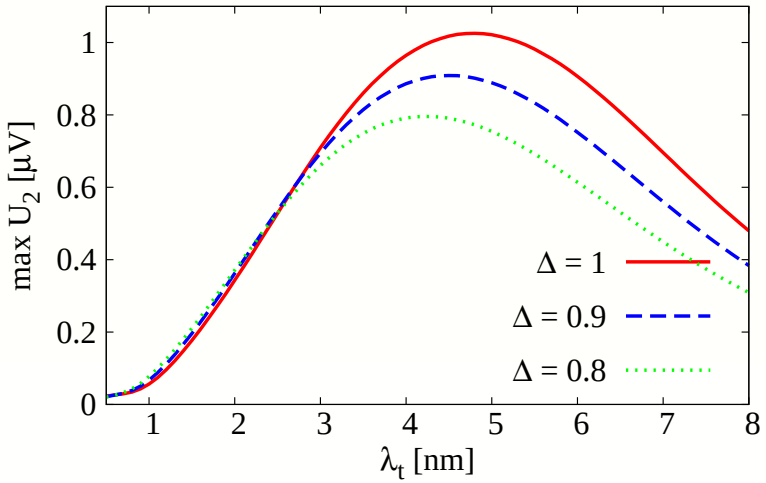
Numerical results: dual spin valve

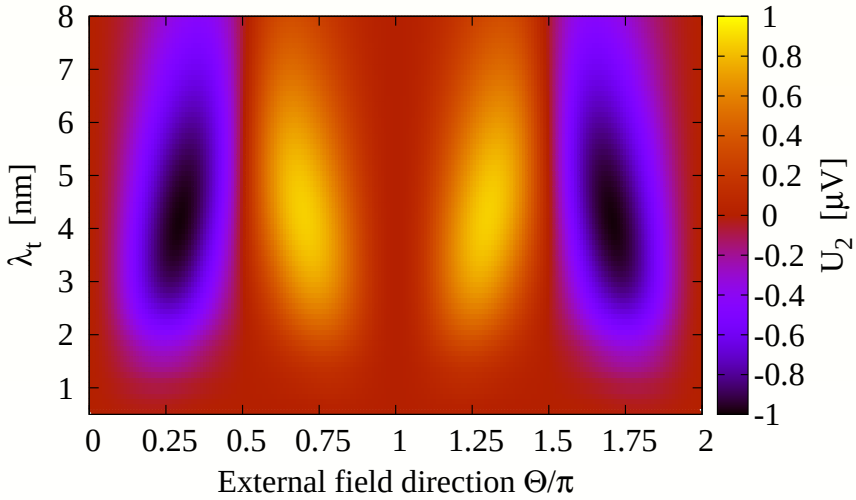
Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

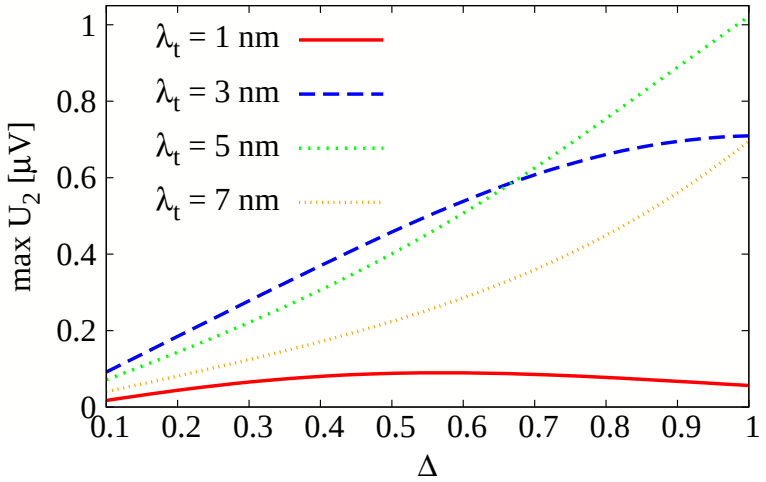


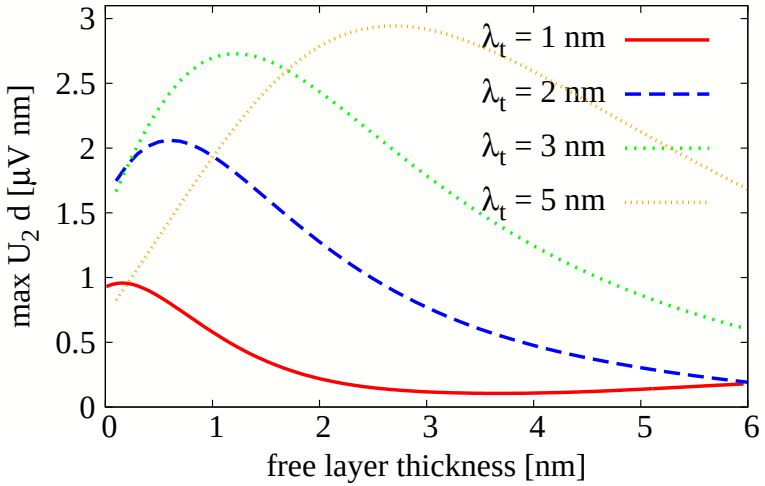
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



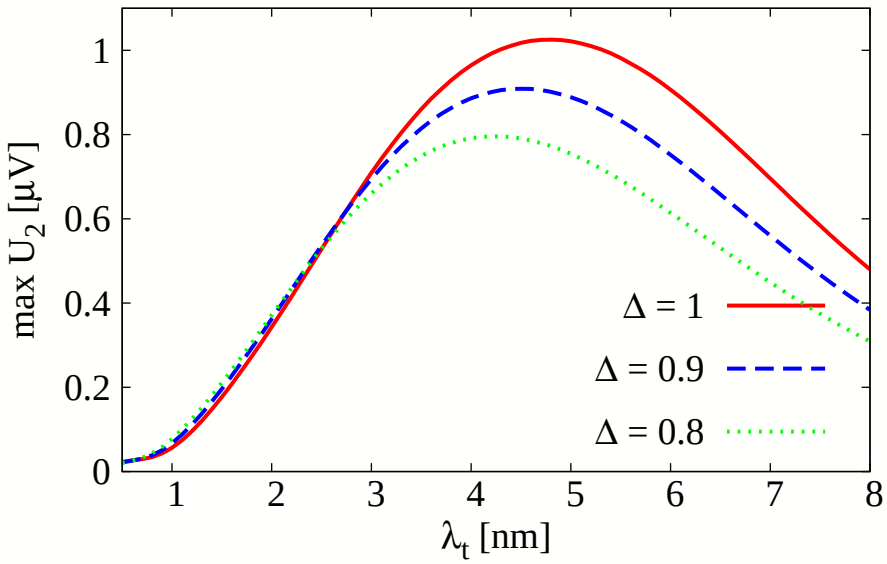


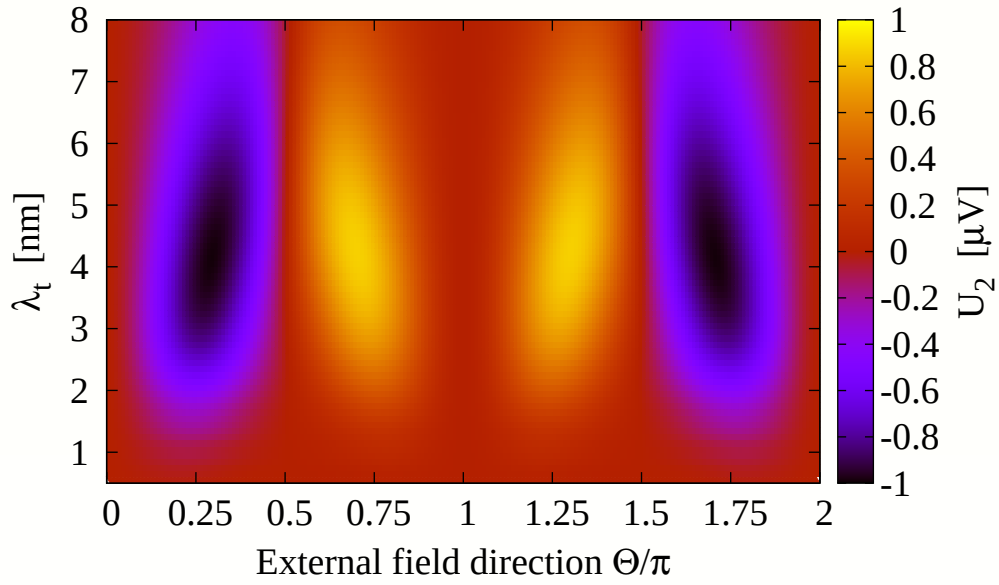


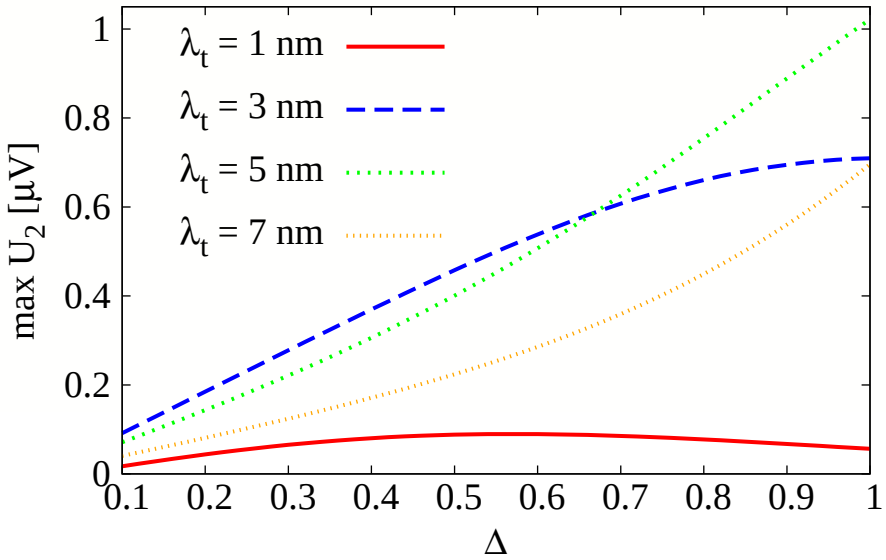


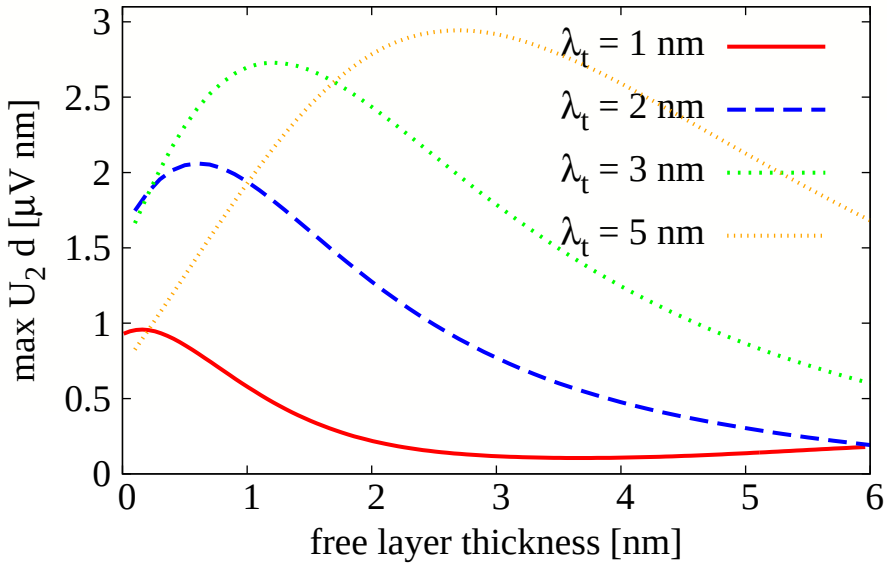
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



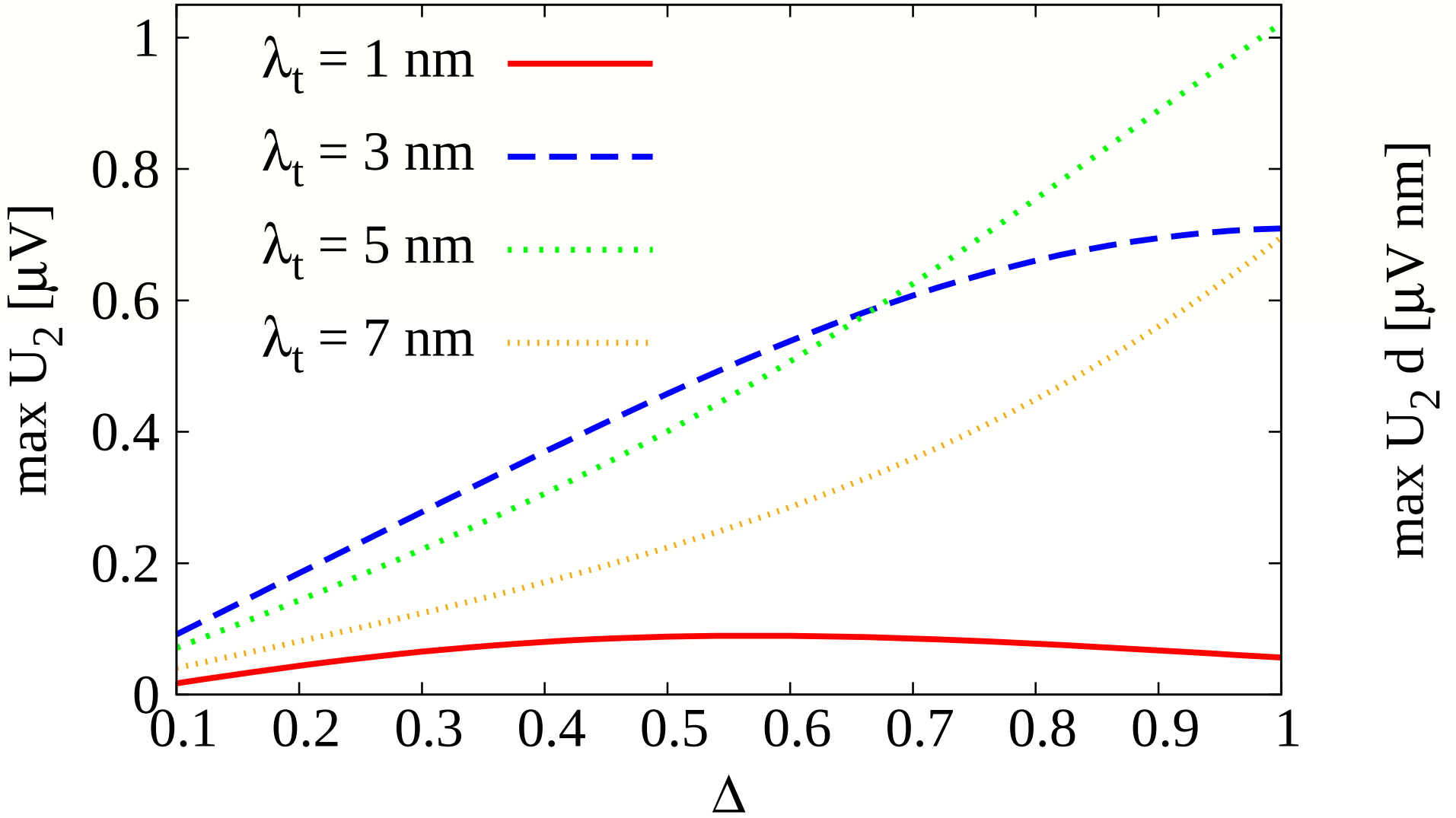






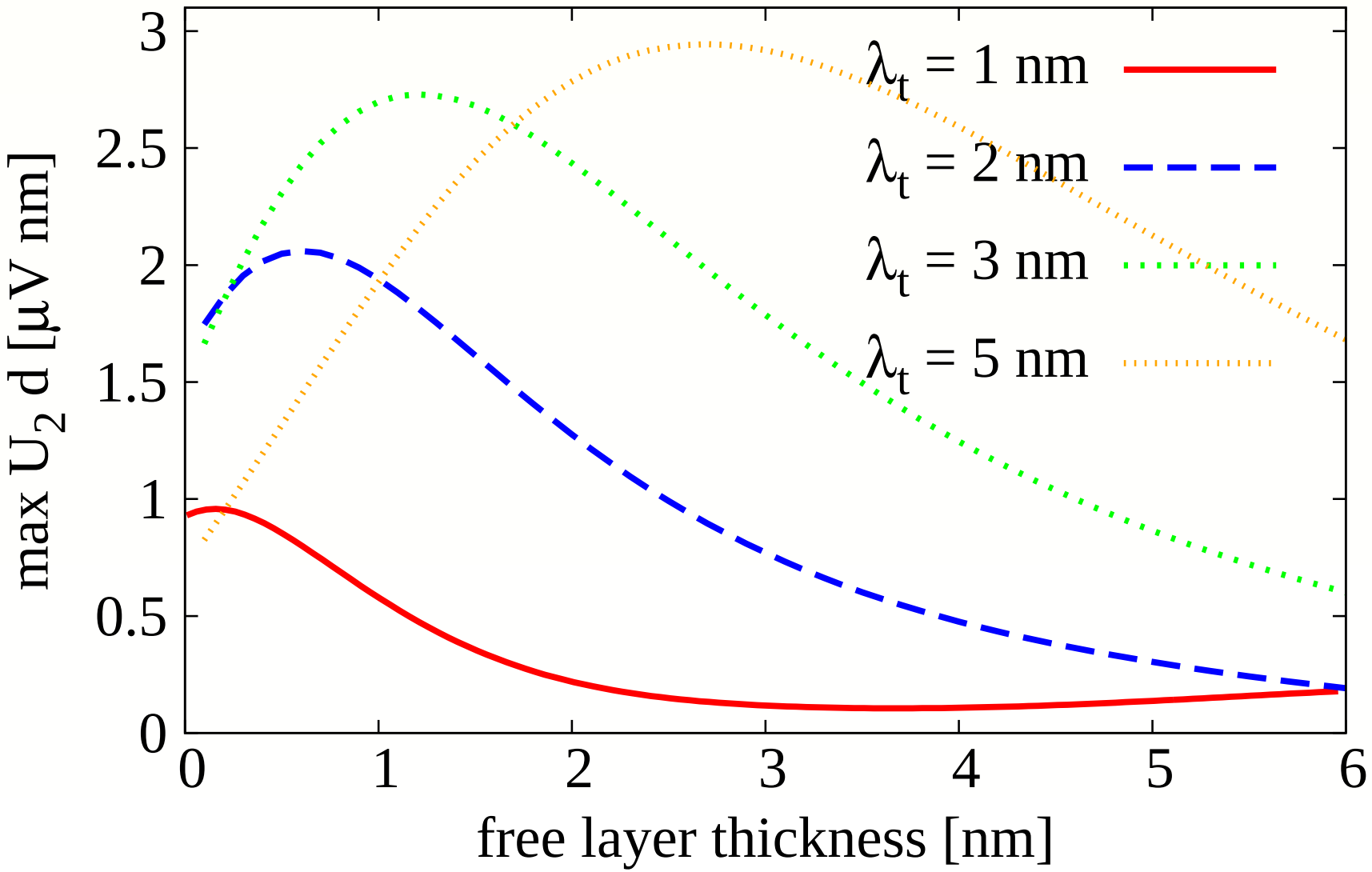
Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



Numerical results: dual spin valve

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



Outline

Conclusions

- A model of second harmonic response to the spin transfer torque has been presented
- Nonlinear behaviour in the second harmonic voltage as a function of λ_t has been observed
- An existence of optimal free layer's width for certain value of λ_t has been reported

SWISS CONTRIBUTION

NANOSPIN

Thank you for your attention

AGH

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

IFM PAN