SWISS CONTRIBUTION

NANOSPIN

Current-induced instabilities of composite free layer with antiferromagnetic interlayer coupling

Pavel Baláž and Józef Barnaś

A. Mickiewicz University
Faculty of Physics
Umultowska 85
61-614 Poznań
Poland

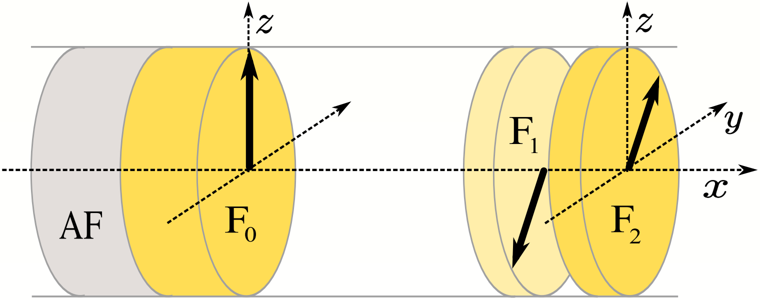
A. Mickiewicz University in Poznań, 25 April 2013

AGH

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Outline

Spinvalve with composite free layer (CFL)



Magnetization dynamics

Landau-Lifshitz-Gilber equation

$$\frac{\mathrm{d}\hat{\boldsymbol{S}}_i}{\mathrm{d}t} + \alpha\,\hat{\boldsymbol{S}}_i \times \frac{\mathrm{d}\hat{\boldsymbol{S}}_i}{\mathrm{d}t} = \boldsymbol{\Gamma}_i\,, \qquad \boldsymbol{\Gamma}_i = -|\gamma_\mathrm{g}|\mu_0\,\hat{\boldsymbol{S}}_i \times \mathbf{H}_{\mathrm{eff}\;i} + \frac{|\gamma_\mathrm{g}|}{M_\mathrm{s}\,d_i}\,\boldsymbol{\tau}_i$$

Effective magnetic field

$$\mathbf{H}_{\mathrm{eff}\;i} = -H_\mathrm{app}\,\hat{\boldsymbol{e}}_z - H_\mathrm{ani}(\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{e}}_z)\,\hat{\boldsymbol{e}}_z + \mathbf{H}_{\mathrm{dem}\;i}(\hat{\boldsymbol{S}}_i) + H_{\mathrm{RKKY}\;i}\,\hat{\boldsymbol{S}}_j$$

Interlayer exchange coupling $H_{\mathrm{RKKY}\;i} = -J_\mathrm{RKKY}/(\mu_0\,M_\mathrm{s}\,d_i)$

Spin transfer torque

$$\boldsymbol{\tau}_{1\parallel} = I\hat{\boldsymbol{S}}_1 \times \left[\hat{\boldsymbol{S}}_1 \times \left(a_1^{(0)}\,\hat{\boldsymbol{S}}_0 + a_1^{(2)}\,\hat{\boldsymbol{S}}_2\right)\right]$$

$$\boldsymbol{\tau}_{1\perp} = I\hat{\boldsymbol{S}}_1 \times \left(b_1^{(0)}\,\hat{\boldsymbol{S}}_0 + b_1^{(2)}\,\hat{\boldsymbol{S}}_2\right)$$

$$\boldsymbol{\tau}_{2\parallel} = Ia_2^{(1)}\,\hat{\boldsymbol{S}}_2 \times \left(\hat{\boldsymbol{S}}_2 \times \hat{\boldsymbol{S}}_1\right)$$

$$\boldsymbol{\tau}_{2\perp} = Ib_2^{(1)}\,\hat{\boldsymbol{S}}_2 \times \hat{\boldsymbol{S}}_1$$

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$
- CFL coordinates: $\tilde{\boldsymbol{S}} = (\theta_1, \phi_1, \theta_2, \phi_2)^{\mathsf{T}}$

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi\, d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$
- CFL coordinates: $\tilde{\boldsymbol{S}} = (\theta_1, \phi_1, \theta_2, \phi_2)^{\mathsf{T}}$
- Landau-Lifshitz-Gilbert equation: $\boxed{\frac{\mathrm{d}\tilde{\boldsymbol{S}}}{\mathrm{d}t} = \bar{\boldsymbol{M}} \cdot \tilde{\boldsymbol{\omega}}}$ where $\bar{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{L}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_2 \end{pmatrix}$, and

  $\boldsymbol{L}_i = \begin{pmatrix} 1 & \alpha \\ -\alpha/\sin\theta_i & 1/\sin\theta_i \end{pmatrix}$, provided that $\alpha^2 \ll 1$.

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$
- CFL coordinates: $\tilde{\boldsymbol{S}} = (\theta_1, \phi_1, \theta_2, \phi_2)^\intercal$
- Landau-Lifshitz-Gilbert equation: $\boxed{\frac{\mathrm{d}\tilde{\boldsymbol{S}}}{\mathrm{d}t} = \bar{\boldsymbol{M}} \cdot \tilde{\boldsymbol{\omega}}}$ where $\bar{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{L}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_2 \end{pmatrix}$, and

  $\boldsymbol{L}_i = \begin{pmatrix} 1 & \alpha \\ -\alpha/\sin\theta_i & 1/\sin\theta_i \end{pmatrix}$, provided that $\alpha^2 \ll 1$.
- Acting torques: $\tilde{\boldsymbol{\omega}} = (\omega_{\theta 1}, \omega_{\phi 1}, \omega_{\theta 2}, \omega_{\phi 2})^\intercal$, where $\omega_{\theta i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\theta i}$ and $\omega_{\phi i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\phi i}$

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$
- CFL coordinates: $\tilde{\boldsymbol{S}} = (\theta_1, \phi_1, \theta_2, \phi_2)^{\mathsf{T}}$
- Landau-Lifshitz-Gilbert equation: $\boxed{\frac{\mathrm{d}\tilde{\boldsymbol{S}}}{\mathrm{d}t} = \bar{\boldsymbol{M}} \cdot \tilde{\boldsymbol{\omega}}}$ where $\bar{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{L}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_2 \end{pmatrix}$, and $\boldsymbol{L}_i = \begin{pmatrix} 1 & \alpha \\ -\alpha/\sin\theta_i & 1/\sin\theta_i \end{pmatrix}$, provided that $\alpha^2 \ll 1$.
- Acting torques: $\tilde{\boldsymbol{\omega}} = (\omega_{\theta 1}, \omega_{\phi 1}, \omega_{\theta 2}, \omega_{\phi 2})^{\mathsf{T}}$, where $\omega_{\theta i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\theta i}$ and $\omega_{\phi i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\phi i}$
- Static points: $\tilde{\boldsymbol{S}}_0$, where $\omega_{\theta i} = 0$ and $\omega_{\phi i} = 0$ for $i = 1, 2$

Linearized Landau-Lifshitz-Gilbert equation

We simplify the model by using $d_1 = \xi d$, where $d = d_2$. Then $H_{\mathrm{RKKY}\,2} = H_{\mathrm{RKKY}}$ and $H_{\mathrm{RKKY}\,1} = \xi^{-1} H_{\mathrm{RKKY}}$.

- Local coordinates: $\hat{\boldsymbol{e}}_{\phi i} = (\hat{\boldsymbol{e}}_z \times \hat{\boldsymbol{S}}_i)/\sin\theta_i$ and $\hat{\boldsymbol{e}}_{\theta i} = \hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{e}}_{\phi i}$, where $i = 1, 2$
- CFL coordinates: $\tilde{\boldsymbol{S}} = (\theta_1, \phi_1, \theta_2, \phi_2)^\mathsf{T}$
- Landau-Lifshitz-Gilbert equation: $\boxed{\dfrac{\mathrm{d}\tilde{\boldsymbol{S}}}{\mathrm{d}t} = \bar{\boldsymbol{M}} \cdot \tilde{\boldsymbol{\omega}}}$ where $\bar{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{L}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}_2 \end{pmatrix}$, and $\boldsymbol{L}_i = \begin{pmatrix} 1 & \alpha \\ -\alpha/\sin\theta_i & 1/\sin\theta_i \end{pmatrix}$, provided that $\alpha^2 \ll 1$.
- Acting torques: $\tilde{\boldsymbol{\omega}} = (\omega_{\theta 1}, \omega_{\phi 1}, \omega_{\theta 2}, \omega_{\phi 2})^\mathsf{T}$, where $\omega_{\theta i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\theta i}$ and $\omega_{\phi i} = \boldsymbol{\Gamma}_i \cdot \hat{\boldsymbol{e}}_{\phi i}$
- Static points: $\tilde{\boldsymbol{S}}_0$, where $\omega_{\theta i} = 0$ and $\omega_{\phi i} = 0$ for $i = 1, 2$
- Linearized LLG in the vicinity of $\tilde{\boldsymbol{S}}_0$: $\boxed{\dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\delta}\tilde{\boldsymbol{S}} = \bar{\boldsymbol{D}} \cdot \boldsymbol{\delta}\tilde{\boldsymbol{S}}}$ where $\boldsymbol{\delta}\tilde{\boldsymbol{S}}(t) = \tilde{\boldsymbol{S}}(t) - \tilde{\boldsymbol{S}}_0$ and $\bar{\boldsymbol{D}} = \bar{\boldsymbol{M}} \cdot \bar{\boldsymbol{J}}$ is the dynamic matrix with $J_{ij} = \partial\tilde{\omega}_i / \partial\tilde{S}_j$.

Dynamic matrix

for $\theta_1 = \theta_2 = \pi/2$: $\bar{\boldsymbol{D}} = \begin{pmatrix} \boldsymbol{D}_1 & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{D}_2 \end{pmatrix}$

- *single spin* dynamic matrix: $\boldsymbol{D}_i = \begin{pmatrix} \omega_i^\tau - \alpha\,\omega_i^{h1} & \alpha\,\omega_i^\tau - \omega_i^{h2} \\ -\alpha\,\omega_i^\tau - \omega_i^{h1} & \omega_i^\tau + \alpha\,\omega_i^{h2} \end{pmatrix}$
- coupling between the free layer's: $\boldsymbol{C}_{ij} = \begin{pmatrix} -\omega_{ij}^\tau + \alpha\,\omega_{Ji} & -\left(\alpha\,\omega_{ij}^\tau + \omega_{Ji}\right)\cos\Delta\phi \\ \alpha\,\omega_{ij}^\tau + \omega_{Ji} & -\left(\omega_{ij}^\tau - \alpha\,\omega_{Ji}\right)\cos\Delta\phi \end{pmatrix}$

Effective magnetic field:

$$\omega_i^{h1} = |\gamma_\mathrm{g}|\mu_0\left[\left(H_\mathrm{ani} - H_{iz}^\mathrm{d}\right)\cos^2\phi_i + H_{ix}^\mathrm{d} - H_{iy}^\mathrm{d}\sin^2\phi_i - H_\mathrm{app}\cos\phi_i\right] + \omega_{Ji}\cos\Delta\phi$$

$$\omega_i^{h2} = |\gamma_\mathrm{g}|\mu_0\left[\left(H_\mathrm{ani} + H_{iy}^\mathrm{d} - H_{iz}^\mathrm{d}\right)\cos 2\phi_i + H_\mathrm{app}\cos\phi_i\right] - \omega_{Ji}\cos\Delta\phi$$

with $\Delta\phi = \phi_1 - \phi_2$, and $\omega_{J1} = \omega_J/\xi$ and $\omega_{J2} = \omega_J$, where $\omega_J = |\gamma_\mathrm{g}|\mu_0\,H_\mathrm{RKKY}$

Spin transfer torque:

$\omega_1^\tau = -\omega_{10}^\tau\cos\phi_1 + \omega_{12}^\tau\cos\Delta\phi$, $\omega_2^\tau = \omega_{21}^\tau\cos\Delta\phi$, and $\omega_{ij}^\tau = \dfrac{a_i^{(j)}\,I}{\mu_0\,M_\mathrm{s}\,d_i}$

Stability in the sense of Lyapunov

Consider $\dot{x} = f(x)$ with $x(0) = 0$ and equilibrium x_e. This equilibrium is

Lyapunov stable if $\forall\epsilon > 0$, $\exists\delta = \delta(\epsilon)$ such that $\|x(0) - x_\mathrm{e}\| < \delta$,
then $\|x(t) - x_\mathrm{e}\| < \epsilon$ for every $t \geq 0$.

Asymptotically stable if $\exists\delta$ such that if $\|x(0) - x_\mathrm{e}\| < \delta$,
then $\lim_{t\to\infty}\|x(t) - x_\mathrm{e}\| = 0$.

Lyapunov stability criterion

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\delta}\tilde{\boldsymbol{S}} = \bar{\boldsymbol{D}} \cdot \boldsymbol{\delta}\tilde{\boldsymbol{S}} \Longrightarrow \boldsymbol{\delta}\tilde{\boldsymbol{S}}(t) = \exp(\bar{\boldsymbol{D}}t)\, \boldsymbol{\delta}\tilde{\boldsymbol{S}}_0,$$

where $\boldsymbol{\delta}\tilde{\boldsymbol{S}}_0$ is initial deviation from the equilibrium $\tilde{\boldsymbol{S}}_0$

Lyapunov criterion

Static state, $\tilde{S}_0$, is stable if and only if all the eigenvalues of $\bar{D}$ have negative real parts. If one of them becomes positive, the static point is unstable.

- stable node: all the eigenvalues have negative real parts
- unstable node: all the eigenvalues have positive real parts
- saddle point: some of the eigenvalues have positive and some negative real parts

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Its eigenvalues are given by $\det(\bar{\boldsymbol{D}} - \lambda \, \bar{\boldsymbol{I}}) = 0$, which leads to $(a - \lambda)(b - \lambda) - bc = 0$

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Its eigenvalues are given by $\det(\bar{\boldsymbol{D}} - \lambda\,\bar{\boldsymbol{I}}) = 0$, which leads to $(a-\lambda)(b-\lambda) - bc = 0$
- The latter expression can be recast as $\lambda^2 - \lambda\,\mathrm{Tr}\bar{\boldsymbol{D}} + \det\bar{\boldsymbol{D}} = 0$ with solution $\lambda_{1,2} = \frac{\mathrm{Tr}\bar{\boldsymbol{D}}}{2} \pm \sqrt{A}$, where $A = \left(\frac{\mathrm{Tr}\bar{\boldsymbol{D}}}{2}\right)^2 - \det\bar{\boldsymbol{D}}$

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Its eigenvalues are given by $\det(\bar{\boldsymbol{D}} - \lambda\,\bar{\boldsymbol{I}}) = 0$, which leads to $(a-\lambda)(b-\lambda) - bc = 0$
- The latter expression can be recast as $\lambda^2 - \lambda\,\mathrm{Tr}\bar{\boldsymbol{D}} + \det\bar{\boldsymbol{D}} = 0$ with solution $\lambda_{1,2} = \frac{\mathrm{Tr}\bar{\boldsymbol{D}}}{2} \pm \sqrt{A}$, where $A = \left(\frac{\mathrm{Tr}\bar{\boldsymbol{D}}}{2}\right)^2 - \det\bar{\boldsymbol{D}}$
- If $A < 0$, $\sqrt{A}$ is imaginary and the stability condition is $\boxed{\mathrm{Tr}\bar{\boldsymbol{D}} < 0}$

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Its eigenvalues are given by $\det(\bar{\boldsymbol{D}} - \lambda\,\bar{\boldsymbol{I}}) = 0$, which leads to $(a-\lambda)(b-\lambda) - bc = 0$
- The latter expression can be recast as $\lambda^2 - \lambda\,\mathrm{Tr}\,\bar{\boldsymbol{D}} + \det\,\bar{\boldsymbol{D}} = 0$ with solution $\lambda_{1,2} = \frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} \pm \sqrt{A}$, where $A = \left(\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2}\right)^2 - \det\,\bar{\boldsymbol{D}}$
- If $A < 0$, $\sqrt{A}$ is imaginary and the stability condition is $\boxed{\mathrm{Tr}\,\bar{\boldsymbol{D}} < 0}$
- If $A > 0$, $\sqrt{A}$ contributs to the real part. The conditions are $\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} < -\sqrt{A}$ and $\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} < \sqrt{A}$. From this we obtain additional condition $\boxed{\det\,\bar{\boldsymbol{D}} > 0}$.

Example: Application to stability of single spin dynamics

- Conside 2×2 single spin dynamic matrix $\bar{\boldsymbol{D}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Its eigenvalues are given by $\det(\bar{\boldsymbol{D}} - \lambda\,\bar{\boldsymbol{I}}) = 0$, which leads to $(a - \lambda)(b - \lambda) - bc = 0$
- The latter expression can be recast as $\lambda^2 - \lambda\,\mathrm{Tr}\,\bar{\boldsymbol{D}} + \det\,\bar{\boldsymbol{D}} = 0$ with solution $\lambda_{1,2} = \frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} \pm \sqrt{A}$, where $A = \left(\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2}\right)^2 - \det\,\bar{\boldsymbol{D}}$
- If $A < 0$, $\sqrt{A}$ is imaginary and the stability condition is $\boxed{\mathrm{Tr}\,\bar{\boldsymbol{D}} < 0}$
- If $A > 0$, $\sqrt{A}$ contributs to the real part. The conditions are $\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} < -\sqrt{A}$ and $\frac{\mathrm{Tr}\,\bar{\boldsymbol{D}}}{2} < \sqrt{A}$. From this we obtain additional condition $\boxed{\det\,\bar{\boldsymbol{D}} > 0}$.
- Note: This simplification is not valid in higher dimensions $n > 2$.

Routh-Hurwitz theorem

- Consider an n-th degree polynomial $\wp(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n$
- Define a matrix $H_\wp = \begin{pmatrix} a_1 & a_0 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & a_0 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_n \end{pmatrix}$

Routh-Hurwitz theorem

The roots of polynomial $\wp(z)$ have all negative real parts if and only if all the leading principal minors of $H_\wp$ are positive.

$$\Delta_1 = a_1 > 0, \quad \Delta_2 = \det \begin{pmatrix} a_1 & a_0 \\ a_3 & a_2 \end{pmatrix} > 0, \quad \Delta_3 = \det \begin{pmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} > 0, \quad \ldots,$$

$$\Delta_n = a_n \Delta_{n-1} > 0$$

Example: Application to stability of single spin dynamics

- We found that the eigenvalues of 2×2 dynamic matrix obeys $\lambda^2 - \lambda \, \mathrm{Tr} \bar{\boldsymbol{D}} + \det \bar{\boldsymbol{D}} = 0$
- The characteristic polynomial is $\wp(\lambda) = a_0 \, \lambda^2 + a_1 \, \lambda + a_0$, where $a_0 = 1$, $a_1 = -\mathrm{Tr} \bar{\boldsymbol{D}}$, $a_2 = \det \bar{\boldsymbol{D}}$.
- We define the matrix $H_\wp = \begin{pmatrix} a_1 & a_0 \\ 0 & a_2 \end{pmatrix}$
- The first stability condition reads $\Delta_1 = a_1 = -\mathrm{Tr} \bar{\boldsymbol{D}} > 0$.

  This leads to $\boxed{\mathrm{Tr} \bar{\boldsymbol{D}} < 0}$.
- The second one $\Delta_2 = \det \begin{pmatrix} a_1 & a_0 \\ 0 & a_2 \end{pmatrix} > 0$. It means that $\mathrm{Tr} \bar{\boldsymbol{D}} \cdot \det \bar{\boldsymbol{D}} < 0$, which leads to $\boxed{\det \bar{\boldsymbol{D}} > 0}$ since $\mathrm{Tr} \bar{\boldsymbol{D}} < 0$.

Application of the Routh-Hurwitz method to CFL

Characteristic polynomial of the dynamic matix: $P(\lambda) = c_4\lambda^4 + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0$

We define matrix $H_P = \begin{pmatrix} c_1 & c_0 & 0 & 0 \\ c_3 & c_2 & c_1 & c_0 \\ 0 & c_4 & c_3 & c_2 \\ 0 & 0 & 0 & c_4 \end{pmatrix}$

Stability conditions

$$\Delta_1 = c_1 > 0, \quad \Delta_2 = \det \begin{pmatrix} c_1 & c_0 \\ c_3 & c_2 \end{pmatrix} > 0, \quad \Delta_3 = \det \begin{pmatrix} c_1 & c_0 & 0 \\ c_3 & c_2 & c_1 \\ 0 & c_4 & c_3 \end{pmatrix} > 0$$

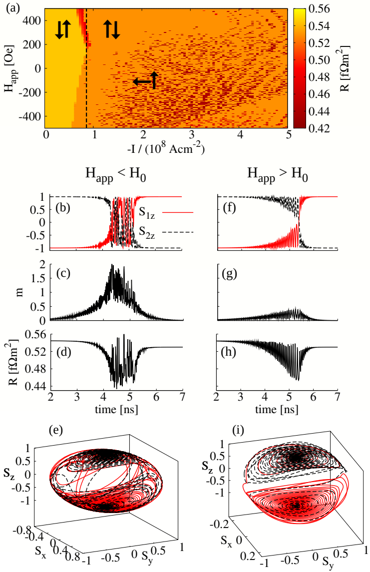
Note: Generally, also condition $\Delta_4 = c_4\Delta_3 > 0$ must be obeyed. However in our case $c_4 = 1$ and, therefore, $\Delta_4 \equiv \Delta_3$.

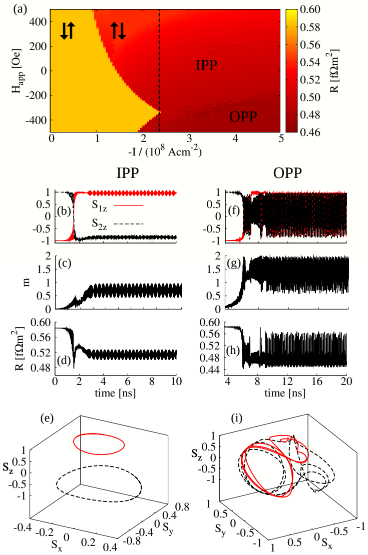
Outline

Simulated spin valve

Cu–IrMn(10)/Py(8)/Cu(8)/Co(ξd)/Ru(1)/Co(d)–Cu

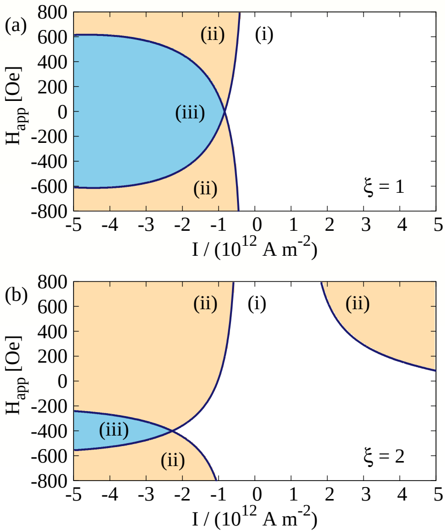
- When $\xi = 1$ we have synthetic antiferromagnet
- When $\xi > 1$ we have synthetic ferrimagnet


(a)
400
200
0
-200
-400
H_{app} [Oe]
0
1
2
3
4
5
-I / (10^8 Acm^{-2})
0.56
0.54
0.52
0.50
0.48
0.46
0.44
0.42
R [fΩm^2]
$H_{app} < H_0$
$H_{app} > H_0$
(b)
(f)
S_{1z}
S_{2z}
(c)
(g)
m
(d)
(h)
R [fΩm^2]
time [ns]
time [ns]
(e)
(i)
S_z
S_x
S_y


(a)
IPP
OPP
H_{app} [Oe]
-I / (10^8 Acm^{-2})
R [fΩm^2]
IPP
OPP
(b)
S_{1z}
S_{2z}
(c)
m
(d)
R [fΩm^2]
time [ns]
(f)
(g)
(h)
time [ns]
(e)
(i)
S_z
S_x
S_y

Outline

Lyapunov exponents for AP configuration



(a) Synthetic antiferromagnet ($\xi = 1$)

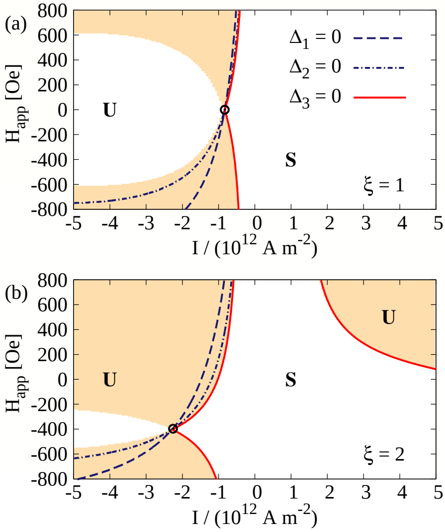
(b) Synthetic ferrimagnet ($\xi = 2$)

(i) AP configuration is table

(ii) two lyapunov exponents are positive

(iii) all four lyapunov exponents are positive

Hurwitz determinants for AP configuration



(a) Synthetic antiferromagnet ($\xi = 1$)

(b) Synthetic ferrimagnet ($\xi = 2$)

- in **S** area all the HDs are positive
- Δ_3 is the first HD which becomes negative
- lines Δ_i for $i = 1, 2, 3$ intersects in the same point

$$\Delta_1 = 0 \Longrightarrow c_1 = 0$$
$$\Delta_2 = 0 \Longrightarrow c_0\, c_3 = 0$$
$$\Delta_3 = c_0\, c_3^2$$

We observe that

- The critical current can be obtained from the condition

$$\Delta_3 = 0$$

- The maximum critical current (cusp point) in the diagram can be obtained solving

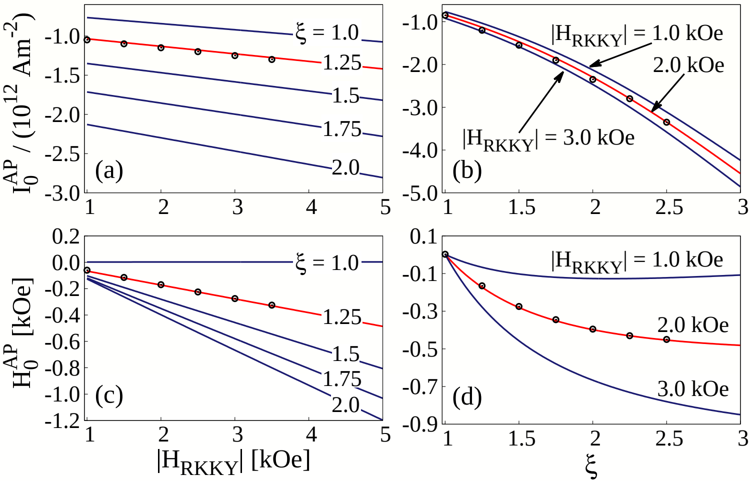
$$\Delta_1 = 0 \quad \text{and} \quad \Delta_2 = 0$$

$\Delta_1 = 0 \Longrightarrow c_1 = 0 \Longrightarrow c_0\, c_3 = 0.$
Therefore $c_0 = 0$ or $c_3 = 0$.
From the second condition we obtain

Critical current density in AP configuration

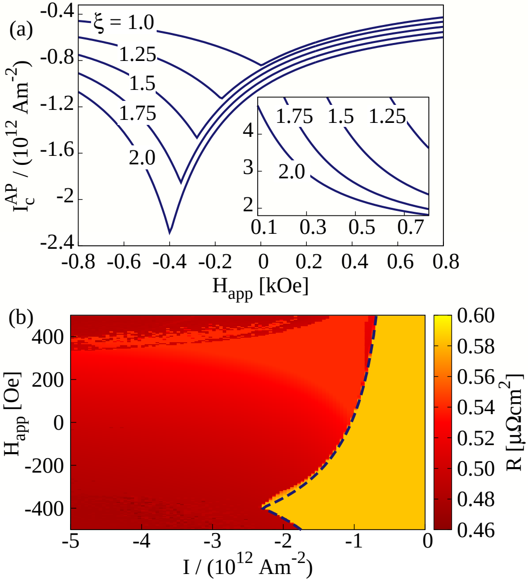
$$I_0^{\mathrm{AP}} = -\alpha \frac{\mu_0 M_{\mathrm{s}}\, \xi\, d}{a_1^{(0)} + a_1^{(2)} + \xi\, a_2^{(1)}} \left[2\, H_{\mathrm{ani}} + H_1^{\mathrm{d}} + H_2^{\mathrm{d}} - (1 + \xi^{-1})\, H_{\mathrm{RKKY}}\right]$$

Inserting I_0^{AP} into $\Delta_1 = 0$ one can obtain the field of intersection, H_0^{AP}.

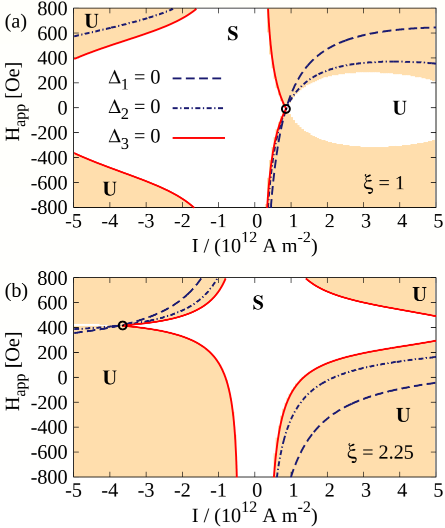


I_0^{AP} / (10^{12} Am^{-2})
-1.0
-1.5
-2.0
-2.5
-3.0
ξ = 1.0
1.25
1.5
1.75
2.0
(a)
1
2
3
4
5
-1.0
-2.0
-3.0
-4.0
-5.0
|H_RKKY| = 1.0 kOe
2.0 kOe
|H_RKKY| = 3.0 kOe
(b)
1
1.5
2
2.5
3
H_0^{AP} [kOe]
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
ξ = 1.0
1.25
1.5
1.75
2.0
(c)
|H_RKKY| [kOe]
0.1
-0.1
-0.3
-0.5
-0.7
-0.9
|H_RKKY| = 1.0 kOe
2.0 kOe
3.0 kOe
(d)
ξ

Critical current density and comparison with numerical simulations



Hurwitz determinants for P configuration



(a) Synthetic antiferromagnet ($\xi = 1$)

(b) Synthetic ferrimagnet ($\xi = 2$)

Critical current density for the P configuration

Critical current density

$$I_0^{\mathrm{P}} = \alpha \, \frac{\mu_0 M_{\mathrm{s}} \, \xi \, d}{a_1^{(0)} - a_1^{(2)} - \xi \, a_2^{(1)}} \left[2 \, H_{\mathrm{ani}} + H_1^{\mathrm{d}} + H_2^{\mathrm{d}} - (1 + \xi^{-1}) \, H_{\mathrm{RKKY}} \right]$$

Critical current density for the P configuration

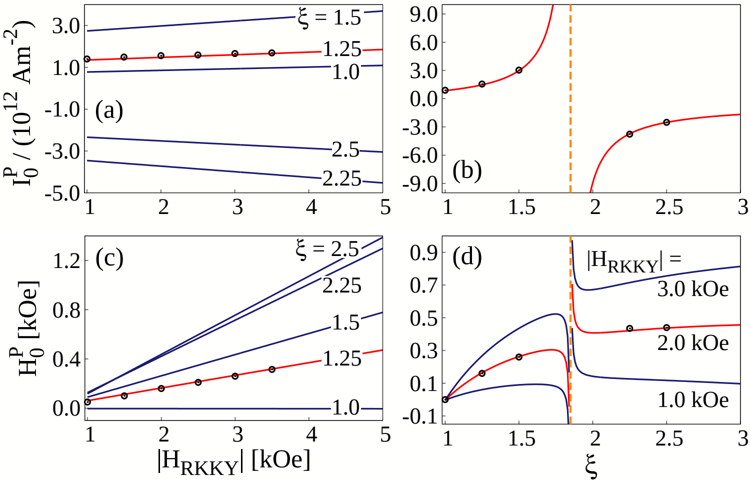
Critical current density

$$I_0^{\mathrm{P}} = \alpha \, \frac{\mu_0 M_{\mathrm{s}} \, \xi \, d}{a_1^{(0)} - a_1^{(2)} - \xi \, a_2^{(1)}} \left[2 \, H_{\mathrm{ani}} + H_1^{\mathrm{d}} + H_2^{\mathrm{d}} - (1 + \xi^{-1}) \, H_{\mathrm{RKKY}} \right]$$

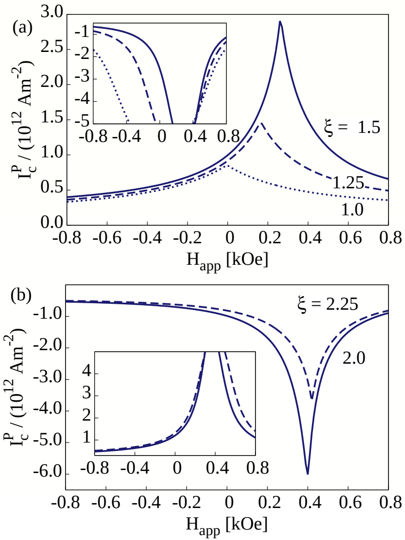
Critical CFL asymmetry, ξ_{c}

$$a_1^{(0)}(\xi_{\mathrm{c}}) - a_1^{(2)}(\xi_{\mathrm{c}}) - \xi_c \, a_2^{(1)}(\xi_{\mathrm{c}}) = 0$$

We obtain $\xi_{\mathrm{c}} \simeq 1.85$


ξ = 1.5
1.25
1.0
(a)
2.5
2.25
3.0
1.0
-1.0
-3.0
-5.0
1
2
3
4
5
I$_0^P$ / (10^{12} Am^{-2})
9.0
6.0
3.0
0.0
-3.0
-6.0
-9.0
(b)
1
1.5
2
2.5
3
(c)
ξ = 2.5
2.25
1.5
1.25
1.0
1.2
0.8
0.4
0.0
H$_0^P$ [kOe]
|H$_{RKKY}$| [kOe]
(d)
|H$_{RKKY}$| =
3.0 kOe
2.0 kOe
1.0 kOe
0.9
0.7
0.5
0.3
0.1
-0.1
ξ

Critical current density



Outline

Conclusions

- We derived expressions for for the cusp point, of the critical current densities in P and AP configurations. These currents are connected as $\frac{I_0^{\mathrm{P}}}{I_0^{\mathrm{AP}}} = -\frac{a_1^{(0)}[\mathrm{AP}] + a_1^{(2)}[\mathrm{AP}] + \xi\, a_2^{(1)}[\mathrm{AP}]}{a_1^{(0)}[\mathrm{P}] - a_1^{(2)}[\mathrm{P}] - \xi\, a_2^{(1)}[\mathrm{P}]}$.
- We shown importance of the internal STT in CFL. Neglecting these STT components leads to $\propto 18\%$ reduction of the critical current when $\xi = 1$, which is quite a significant amount.
- We found single condition for the critical current density, $\Delta_3 = 0$, which holds for all studied ξ and H_{app}.

SWISS CONTRIBUTION

NANOSPIN

Thank you for your attention

AGH

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE