

Spin dependent transport and current-induced magnetization dynamics in metallic spin valves

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Universität Duisburg-Essen, 4 December 2012



Nanoscale spin torque devices for spin electronics

Joint research project under the framework of Polish-Swiss research programme



nanospin.agh.edu.pl



Partners

- AGH University of Science and Technology in Kraków
T. Stobiecki – coordinator
- Institute of Molecular Physics in Poznań, Polish Academy of Sciences
J. Dubowik – experiment, J. Barnaś – theory
- Ecolé Polytechnique Fédérale in Lausanne
J.-Ph. Ansermet

Objectives

jointly developing novel **nanoscale spintronic devices** based on the **spin transfer torque effect**, which promises unrivaled future scaling, flexibility and low power consumption



www.zfmezo.amu.edu.pl

Head: prof. Józef Barnaś

Research activities

- Single Molecular Magnets and Adatoms
- Kondo effect in quantum Dots
- Quantum and Spin Hall Effect
- Electronic transport in graphene
- Current-induced dynamics in magnetic spin valves
- Current-induced domain wall motion

Recent projects

- European Marie Currie Research Training Network **SPINSWITCH**
- National Centre for Magnetic Nanostructures **SpinLab**
- Polish-Swiss Research Project **NanoSpin**

with Polish Academy of Sciences, University of Technology Kraków, and École Polytechnique Fédérale de Lausanne

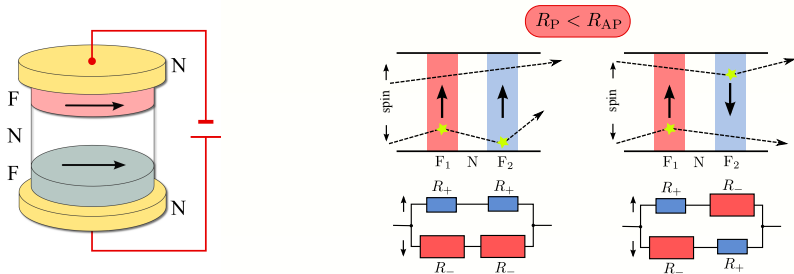


- 1 Introduction
- 2 Spin-torque in metallic spin valves
- 3 Dual spin valve
- 4 Nonlinear magnetoresistance in dual spin valves
- 5 Transverse spin transport
- 6 Transport through an perpendicular polarizer

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Spin valves and Giant magnetoresistance



M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas
Giant Magnetoresistance of (001)Fe/(001)Cr Magnetic Superlattices
 Phys. Rev. Lett. **61**, 2472–2475 (1988)



G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn
Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange
 Phys. Rev. B **39**, 4828–4830 (1989)



R. E. Camley and J. Barnaś
Theory of giant magnetoresistance effects in magnetic layered structures with antiferromagnetic coupling
 Phys. Rev. Lett. **63**, 664–667 (1989)



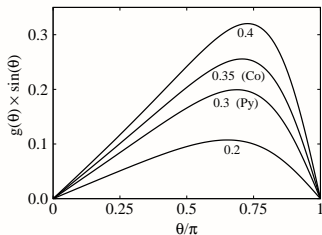
T. Valet and A. Fert
Theory of the perpendicular magnetoresistance in magnetic multilayers
 Phys. Rev. B **48**, 7099–7113 (1993)

Current-induced dynamics and magnetization switching

Magnetization can be **switched by electric current** without need of magnetic field

Slonczewski's model (ballistic)

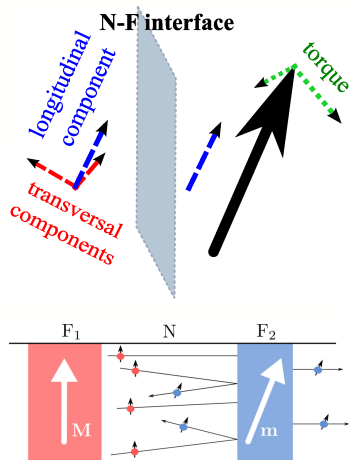
J. Magn. Magn. Mater. **159**, L1-L7 (1996)



$$\tau_{\text{Sloncz}} = \frac{I g(\theta)}{e} \hat{s} \times (\hat{s} \times \hat{S})$$

where

$$g(\theta) = \left[-4 + (1 + P)^3 \frac{3 + \cos \theta}{4P^{3/2}} \right]^{-1}$$

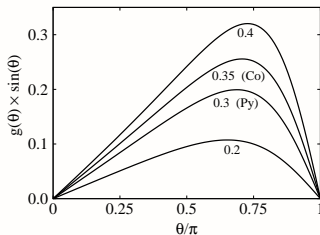


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J. Magn. Magn. Mater. **159**, L1-L7 (1996)



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Unified description (diffusive)

Description of **spin-transfer torque** should be **consistent with** description of **giant magnetoresistance** (Valet-Fert model)



J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev
From giant magnetoresistance to current-induced switching by spin transfer
Phys. Rev. B **72**, 024426 (2005)

Spin-transfer torque

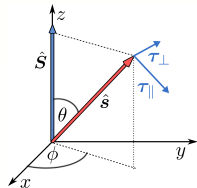
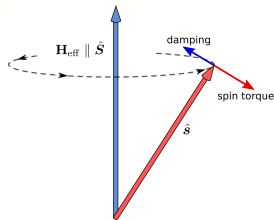
$$\tau_{\theta} = a(\theta) I \hat{s} \times (\hat{s} \times \hat{S})$$

$$\tau_{\phi} = b(\theta) I \hat{s} \times \hat{S}$$

Equation of motion

Landau-Lifshitz-Gilbert equation

$$\frac{d\hat{\mathbf{s}}}{dt} = -|\gamma_g|\mu_0 \hat{\mathbf{s}} \times \mathbf{H}_{\text{eff}} - \alpha \hat{\mathbf{s}} \times \frac{d\hat{\mathbf{s}}}{dt} + \frac{|\gamma_g|}{M_s d} (\tau_\theta + \tau_\phi)$$



Effective magnetic field

$$\mathbf{H}_{\text{eff}} = -H_{\text{ext}}\hat{\mathbf{e}}_z - H_{\text{ani}}(\hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_z)\hat{\mathbf{e}}_z + \mathbf{H}_{\text{demag}}$$

Spin-transfer torque

$$\tau_\theta = a(\theta) I \hat{\mathbf{s}} \times (\hat{\mathbf{s}} \times \hat{\mathbf{S}})$$

$$\tau_\phi = b(\theta) I \hat{\mathbf{s}} \times \hat{\mathbf{S}}$$

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Mathematical description

Two channels model

bulk resistivities

$$\rho_{\uparrow(\downarrow)} = 2\rho^* (1 \mp \beta)$$

interface resistances

$$R_{\uparrow(\downarrow)} = 2R^* (1 \mp \gamma)$$

- β bulk asymmetry parameters
- γ interfacial asymmetry parameter

Diffusive transport

$$\frac{\partial^2 \bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow}}{\partial x^2} = \frac{1}{l_{sf}^2} (\bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow})$$

$$\frac{\partial^2 \bar{\mu}_{\uparrow} + \bar{\mu}_{\downarrow}}{\partial x^2} = \eta \frac{\partial^2 (\bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow})}{\partial x^2}$$



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for electrochemical potentials we get

$$\bar{\mu}_{\uparrow} = (1 + \eta) \left[Ae^{x/l_{sf}} + Be^{-x/l_{sf}} \right] + Cx + G$$

$$\bar{\mu}_{\downarrow} = (\eta - 1) \left[Ae^{x/l_{sf}} + Be^{-x/l_{sf}} \right] + Cx + G$$



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Mathematical description

Magnetic layer

$$\check{\mu} = \bar{\mu}_0 \check{1} + g \check{\sigma}_z$$

$$\bar{\mu}_0 = \frac{\bar{\mu}_\uparrow + \bar{\mu}_\downarrow}{2}, \quad g = \frac{\bar{\mu}_\uparrow - \bar{\mu}_\downarrow}{2}$$

with g being **spin accumulation**

$$\check{j} = -\rho(E_F) \check{D} \frac{\partial \check{\mu}}{\partial x}$$

$$\check{j} = \frac{1}{2} (j_0 \check{1} + j_z \check{\sigma}_z)$$

with j_z being **spin current**

Diffusive transport

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Phys. Rev. B **72**, 024426 (2005)

Mathematical description

Nonmagnetic layer

has no natural quantization axis

$$\begin{aligned}\check{\mu} &= \bar{\mu}_0 \check{\mathbf{I}} + \mathbf{g} \cdot \check{\sigma} \\ \check{\mathbf{j}} &= \frac{1}{2} (j_0 \check{\mathbf{I}} + \mathbf{j} \cdot \check{\sigma})\end{aligned}$$

where

$$\mathbf{g} = (g_x, g_y, g_z), \quad \mathbf{j} = (j_x, j_y, j_z)$$

are 3D vectors written in the coordinate system of one of the adjacent magnetic layers

Diffusive transport

$$\begin{aligned}\frac{\partial^2 \bar{\mu}_\uparrow - \bar{\mu}_\downarrow}{\partial x^2} &= \frac{1}{l_{\text{sf}}^2} (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow) \\ \frac{\partial^2 \bar{\mu}_\uparrow + \bar{\mu}_\downarrow}{\partial x^2} &= \eta \frac{\partial^2 (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)}{\partial x^2}\end{aligned}$$

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Boundary conditions at N/F interface

- **particle current** is continuous across all interfaces

$$e^2 j_0 = (G_{\uparrow} + G_{\downarrow})(\bar{\mu}_0^F - \bar{\mu}_0^N) + (G_{\uparrow} - G_{\downarrow})(g_z^F - g_z^N)$$

- **spin current component parallel to the magnetization** is continuous across the interface

$$e^2 j_z = (G_{\uparrow} - G_{\downarrow})(\bar{\mu}_0^F - \bar{\mu}_0^N) + (G_{\uparrow} + G_{\downarrow})(g_z^F - g_z^N)$$

- **transversal component of the spin current** vanishes in the magnetic layer – jump at the interface

$$e^2 j_x = -2\text{Re}\{G_{\uparrow\downarrow}\}g_x^N + 2\text{Im}\{G_{\uparrow\downarrow}\}g_y^N$$

$$e^2 j_y = -2\text{Re}\{G_{\uparrow\downarrow}\}g_y^N - 2\text{Im}\{G_{\uparrow\downarrow}\}g_x^N$$



A. Brataas, Yu.V. Nazarov, G.E.W. Bauer

Spin-transport in multi-terminal normal metal-ferromagnet systems with non-collinear magnetizations
 Eur. Phys. J. B **22**, 99 (2001)

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Spin-transfer torque

$$\tau = \frac{\hbar}{2}(\mathbf{j}_{\perp L} - \mathbf{j}_{\perp R})$$

Boundary conditions at N/F interface

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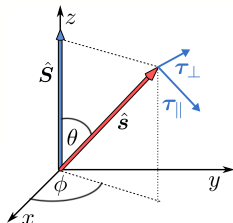
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Components

in-plane

$$\tau_{\theta} = -\frac{\hbar}{2} j_y' |_{N/F}$$

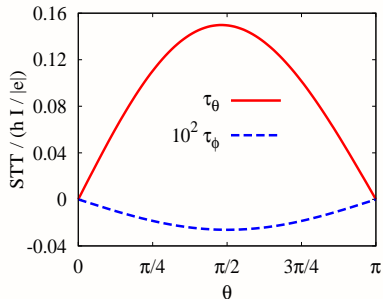
out-of-plane

$$\tau_{\phi} = \frac{\hbar}{2} j_x' |_{N/F}$$

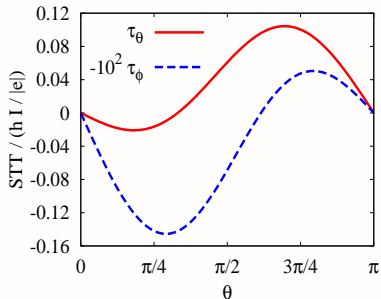
Results

Calculations for real structures

Standard spin valve Py(20)/Cu(10)/Py(8)



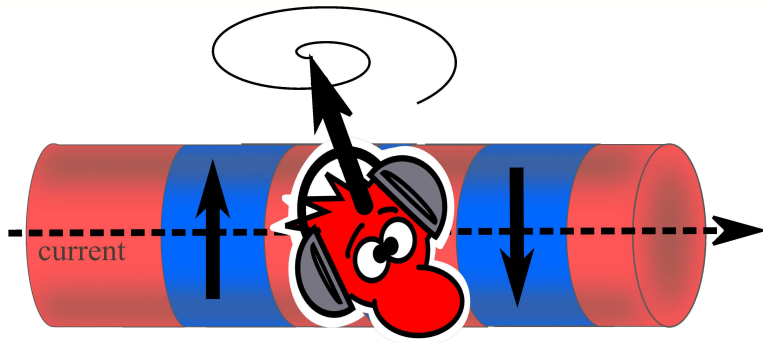
Nonstandard spin valve Co(8)/Cu(10)/Py(8)



Outline

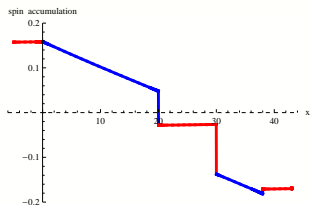
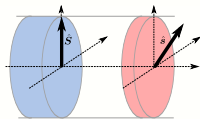
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What is a dual spin valve?



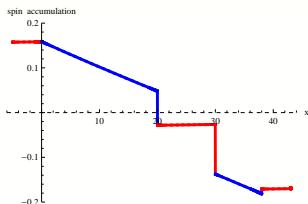
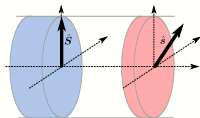
Spin accumulation in dual spin valve

spin accumulation in single spin valve

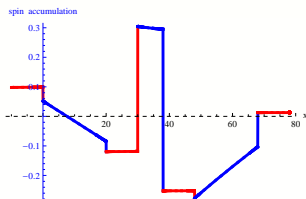
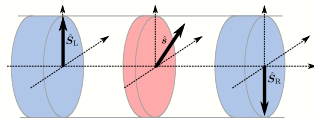


Spin accumulation in dual spin valve

spin accumulation in single spin valve



spin accumulation in dual spin valve



L. Berger

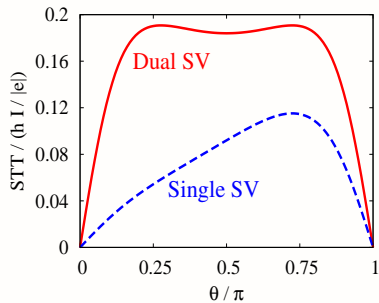
*Multilayer Configuration for Experiments of Spin Precession Induced by a DC Current*J. Appl. Phys. **93**, 7683 (2003)

Enhancement of switching

single vs. dual spin valve

Co(20)/Cu(10)/Co(8) vs. Co(20)/Cu(10)/Co(8)/Cu(10)/Co(20)

Spin-transfer torque

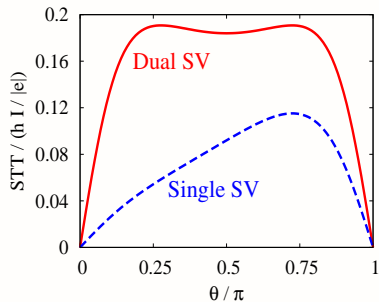


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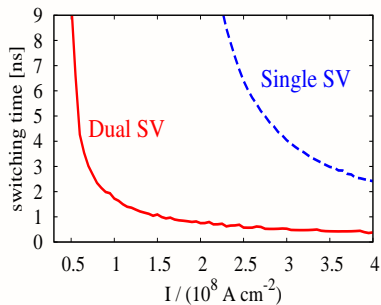
single vs. dual spin valve

Co(20)/Cu(10)/Co(8) vs. Co(20)/Cu(10)/Co(8)/Cu(10)/Co(20)

Spin-transfer torque



Switching time

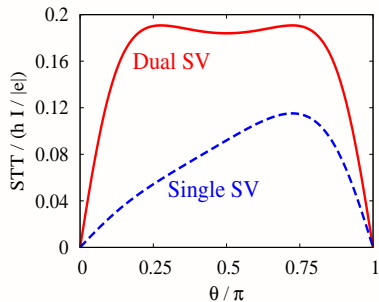


Enhancement of switching

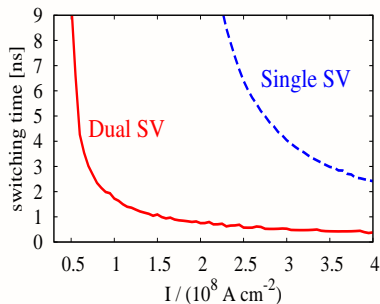
single vs. dual spin valve

Co(20)/Cu(10)/Co(8) vs. Co(20)/Cu(10)/Co(8)/Cu(10)/Co(20)

Spin-transfer torque

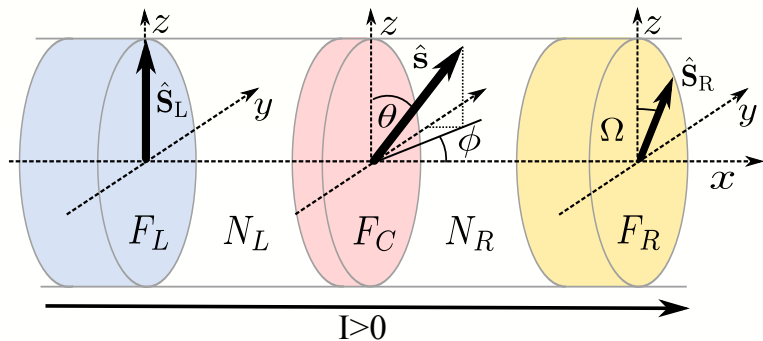


Switching time



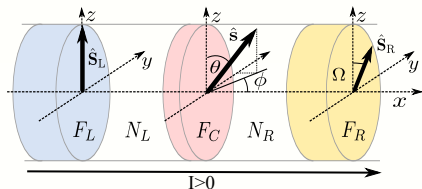
Is this all?

Question: How torque changes in non-collinear configurations?



Noncollinear configurations in dual spin valves

Co(20)/Cu(10)/Py(4)/Cu(4)/Co(10)/IrMn(8)



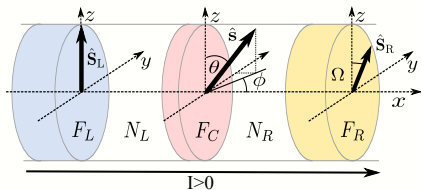
P.B., M. Gmitra, J. Barnaś

Current-induced dynamics in non-collinear dual spin-valves

Phys. Rev. B **80**, 174404 (2009)

Noncollinear configurations in dual spin valves

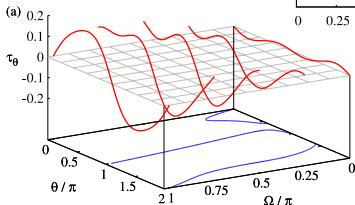
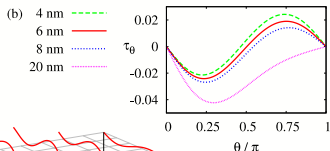
Co(20)/Cu(10)/Py(4)/Cu(4)/Co(10)/IrMn(8)



P.B., M. Gmitra, J. Barnaś

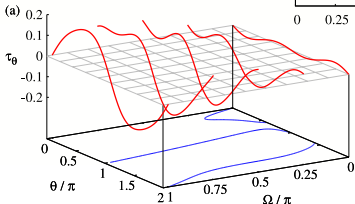
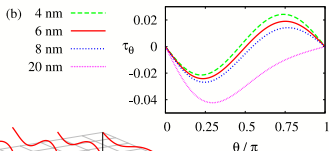
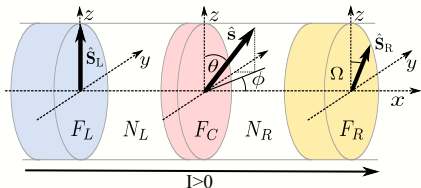
Current-induced dynamics in non-collinear dual spin-valves

Phys. Rev. B **80**, 174404 (2009)



Noncollinear configurations in dual spin valves

Co(20)/Cu(10)/Py(4)/Cu(4)/Co(10)/IrMn(8)



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Current-induced dynamics in non-collinear dual spin-valves

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Current-induced dynamics

We calculated **average**

$$\langle s_z \rangle = \frac{1}{t_{\text{end}} - t_{\text{eq}}} \int_{t_{\text{eq}}}^{t_{\text{end}}} s_z dt$$

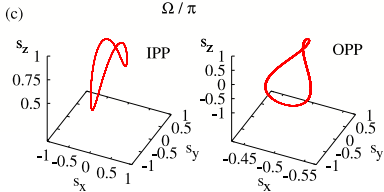
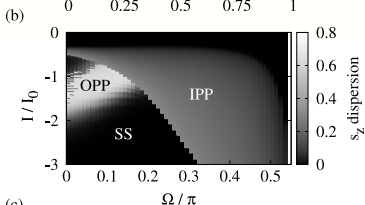
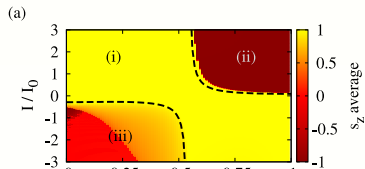
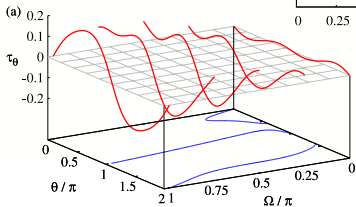
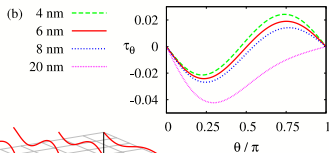
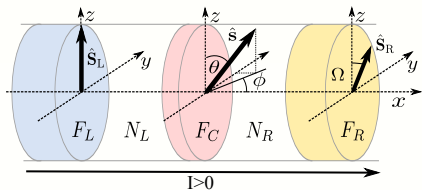
and **dispersion**

$$\mathcal{D}(s_z) = \sqrt{\langle s_z^2 \rangle - \langle s_z \rangle^2}$$

for each couple of I and Ω to map the dynamic behaviour.

Noncollinear configurations in dual spin valves

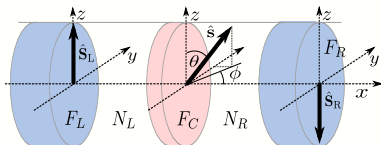
Co(20)/Cu(10)/Py(4)/Cu(4)/Co(10)/IrMn(8)



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Nonlinear magnetoresistance in dual spin valves



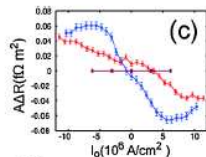
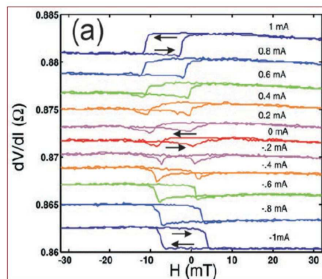
Experimental works



A. Aziz, O. P. Wessely, M. Ali, D. M. Edwards, C. H. Marrows, B. J. Hickey, and M. G. Blamire
Nonlinear giant magnetoresistance in dual spin valves
Phys. Rev. Lett. **103**, 237203 (2009)



N. Banerjee, A. Aziz, M. Ali, J. W. A. Robinson, B. J. Hickey, and M. G. Blamire
Thickness dependence and the role of spin transfer torque in nonlinear giant magnetoresistance of permalloy dual spin valves
Phys. Rev. B **82**, 224402 (2010)



Experimental results [Aziz *et al.*, PRL (2009)]: (b) minor loops for $\text{Co}_{90}\text{Fe}_{10}(6)/\text{Cu}(4)/\text{Py}(1)/\text{Cu}(2)/\text{Co}_{90}\text{Fe}_{10}(6)/\text{IrMn}(10)$ (c) Py thickness: 1 nm (blue), 2 nm (red), and 8 nm (magenta)

Nonlinear magnetoresistance in dual spin valves

Model's assumptions

- Spin accumulation in the central layer changes density of states on Fermi level
- This may change bulk/interfacial material parameters in the central layer

We extended the diffusion transport model



J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev

From giant magnetoresistance to current-induced switching by spin transfer
 Phys. Rev. B **72**, 024426 (2005)

Bulk contribution

$$\rho^* = \rho_0^* + q \langle g \rangle$$

$$\beta = \beta_0 + \xi \langle g \rangle$$

Interfacial contribution

$$R^* = R_0^* + q' g(x_i)$$

$$\gamma = \gamma_0 + \xi' g(x_i)$$

where

- $g(x)$ is spin accumulation
- ρ_0^* and R_0^* are *zero-current* bulk resistivity and interfacial resistance
- β_0 and γ_0 are *zero-current* bulk/interfacial asymmetry parameters
- q, ξ, q', ξ' are phenomenological parameters

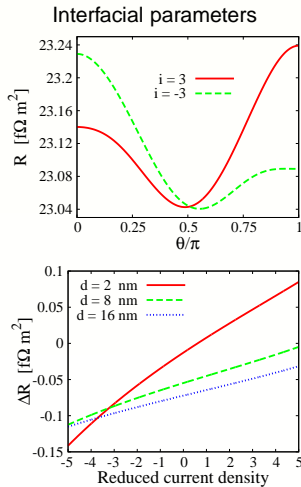
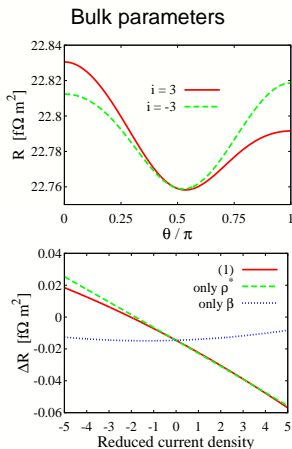


P. Baláz, and J. Barnaś

Nonlinear magnetotransport in dual spin valves
 Phys. Rev. B **82**, 104430 (2010)

Nonlinear magnetoresistance in dual spin vales

Calculations for Cu - Co(6) / Cu(4) / Py(2) / Cu(2) / Co(6) / IrMn(10) - Cu



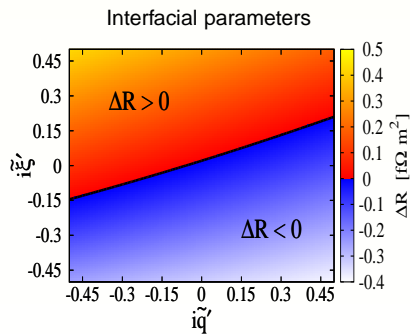
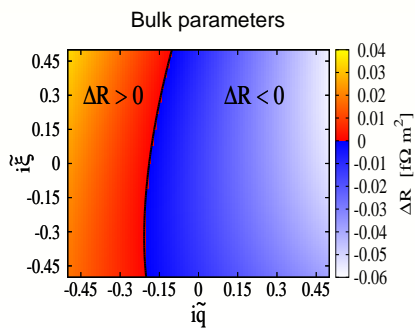
P. Baláz, and J. Barnaš

Nonlinear magnetotransport in dual spin vales
 Phys. Rev. B **82**, 104430 (2010)

for $\tilde{q} = 0.1$, $\tilde{\xi} = 0.1$, $I_0 = 10^8 \text{ Acm}^{-2}$

Nonlinear magnetoresistance in dual spin valves

Calculations for Cu - Co(6) / Cu(4) / Py(2) / Cu(2) / Co(6) / IrMn(10) - Cu



for **interfaces** ΔR is **symmetric** with current density



P. Baláz, and J. Barnaś

Nonlinear magnetotransport in dual spin valves
Phys. Rev. B **82**, 104430 (2010)

Outline

- 1 Introduction
- 2 Spin-torque in metallic spin valves
- 3 Dual spin valve
- 4 Nonlinear magnetoresistance in dual spin valves
- 5 Transverse spin transport**
- 6 Transport through an perpendicular polarizer

Transverse spin diffusion

Motivation



T. Taniguchi, H. Imamura

Spin pumping in ferromagnetic multilayers
Mod. Phys. Lett. B **22**, 2909 – 2929 (2008)



T. Taniguchi, S. Yakata, H. Imamura, Y. Ando

Determination of Penetration Depth of Transverse Spin Current in Ferromagnetic Metals by Spin Pumping
arXiv:0708.3528v3 [cond-mat.mes-hall]



T. Taniguchi, S. Yakata, H. Imamura, Y. Ando

Penetration Depth of Transverse Spin Current in Ferromagnetic Metals
IEEE Trans. Mag. **44**, 2636 (2008); arXiv:0806.3315v1 [cond-mat.mes-hall]

Estimated transverse spin diffusion (penetration) length

- Py 3.7 nm
- CoFe 2.5 nm
- CoFeB 12 nm
- Co 1.7 nm

Transverse spin diffusion

Modified continuity equation

$$\frac{df_0}{dt} = -\frac{\partial j_0}{\partial x},$$

$$\frac{d\mathbf{f}}{dt} = \frac{J}{\hbar} (\hat{\mathbf{S}} \times \mathbf{f}) - \frac{\partial \mathbf{j}}{\partial x} + \frac{\mathbf{f}}{\tau_{\text{sf}}},$$

Current tensor

$$\check{\mathbf{j}} = \begin{pmatrix} j_{\uparrow} & j_{\uparrow\downarrow} \\ j_{\downarrow\uparrow} & j_{\downarrow} \end{pmatrix} = - \begin{pmatrix} D_{\uparrow} \frac{\partial f_{\uparrow}}{\partial x} & D_{\perp} \frac{\partial f_{\uparrow\downarrow}}{\partial x} \\ D_{\perp} \frac{\partial f_{\downarrow\uparrow}}{\partial x} & D_{\downarrow} \frac{\partial f_{\downarrow}}{\partial x} \end{pmatrix}$$

where

$$D_{\uparrow(\downarrow)} = D_0 / (1 \mp \beta)$$

$$D_{\perp} = \frac{D_{\uparrow} + D_{\downarrow}}{2}$$



A. Shpiro, P. Lévy, and S. Zhang

Self-consistent treatment of nonequilibrium spin torques in magnetic multilayers
 Phys. Rev. B **67** 104430 (2002)

Charge and longitudinal components

$$\frac{\partial^2 f_z}{\partial x^2} = \frac{f_z}{\lambda_{\text{sf}}^2}$$

$$\frac{\partial^2 f_0}{\partial x^2} = -\beta \frac{\partial^2 f_z}{\partial x^2}$$

where $\lambda_{\text{sf}} = \sqrt{D_0 \tau_{\text{sf}}}$.

Transverse spin components

$$\frac{\partial^2 f_x}{\partial x^2} = -\frac{f_y}{\lambda_J^2} + \frac{f_x}{\lambda_{\perp}^2}$$

$$\frac{\partial^2 f_y}{\partial x^2} = \frac{f_x}{\lambda_J^2} + \frac{f_y}{\lambda_{\perp}^2}$$

where $\lambda_J = \sqrt{\hbar D_{\perp} / J}$ and $\lambda_{\perp} = \sqrt{D_{\perp} \tau_{\text{sf}}}$

Solution for the transver components

Longitudinal spin accumulation

$$\mu_z(x) = A e^{x/\lambda_{sf}} + B e^{-x/\lambda_{sf}}$$

- λ_{sf} : spin diffusion length

Transverse spin accumulation

$$\mu_x(x) = C_1 e^{x/l_-} + C_2 e^{-x/l_-} + C_3 e^{x/l_+} + C_4 e^{-x/l_+}$$

$$\frac{1}{l_{\pm}} = \frac{1}{\lambda_t} \pm i \frac{1}{\lambda_p}$$

- λ_t : transverse decay length
- λ_p : precessional length

we define $\Delta = \lambda_t/\lambda_p$, which is $\Delta < 1$

$$\frac{1}{l_{\pm}} = \frac{1 \pm i\Delta}{\lambda_t}$$

Solution for the transverse components

Longitudinal charge and spin current

$$\frac{1}{\rho(E_F)} j_0 = -2C \frac{D_0}{1-\beta^2}$$

$$\frac{1}{\rho(E_F)} j_z(x) = \frac{\beta}{\rho(E_F)} j_0 - \frac{2D_0}{\lambda_{sf}} \left[A e^{x/\lambda_{sf}} - B e^{-x/\lambda_{sf}} \right]$$

Transverse spin current

$$\frac{1}{\rho(E_F)} j_{x(y)} = \pm \frac{D_{\perp}}{\lambda_t} \left\{ \left[(\Delta A_1 - B_1) \sin\left(\frac{\Delta x}{\lambda_t}\right) \mp (A_1 + \Delta B_1) \cos\left(\frac{\Delta x}{\lambda_t}\right) \right] e^{x/\lambda_t} \pm \right.$$

$$\left. \left[(\Delta A_2 + B_2) \sin\left(\frac{\Delta x}{\lambda_t}\right) \pm (A_2 - \Delta B_2) \cos\left(\frac{\Delta x}{\lambda_t}\right) \right] e^{-x/\lambda_t} \right\}$$

Boundary conditions at the N/F interfaces

Longitudinal components

$$e^2 j_0 = (G_{\uparrow} + G_{\downarrow})(\mu_0^F - \mu_0^N) + (G_{\uparrow} - G_{\downarrow})(\mu_z^F - \mu_z^N)$$

$$e^2 j_z = (G_{\uparrow} - G_{\downarrow})(\mu_0^F - \mu_0^N) + (G_{\uparrow} + G_{\downarrow})(\mu_z^F - \mu_z^N)$$

Transversal components

$$e^2 j_x = 2 \operatorname{Re} \{ G_{\uparrow\downarrow} \} (\mu_x^F - \mu_x^N) + 2 \operatorname{Im} \{ G_{\uparrow\downarrow} \} (\mu_y^F - \mu_y^N)$$

$$e^2 j_y = 2 \operatorname{Re} \{ G_{\uparrow\downarrow} \} (\mu_y^F - \mu_y^N) - 2 \operatorname{Im} \{ G_{\uparrow\downarrow} \} (\mu_x^F - \mu_x^N)$$

Current continuity

$$j_x^F = j_x^N = j_x$$

$$j_y^F = j_y^N = j_y$$

$$j_z^F = j_z^N = j_z$$

Spin transfer torque

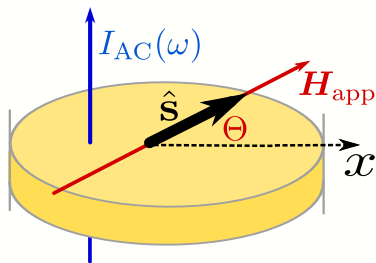
Spin transfer torque

$$\tau_x = \frac{\hbar}{2} \int_{x_L}^{x_R} \frac{\partial j_x}{\partial x} dx = \frac{\hbar}{2} [j_x(x_R) - j_x(x_L)] \equiv \tau_{\perp}$$

$$\tau_y = \frac{\hbar}{2} \int_{x_L}^{x_R} \frac{\partial j_y}{\partial x} dx = \frac{\hbar}{2} [j_y(x_R) - j_y(x_L)] \equiv \tau_{\parallel}$$

where x_L and x_R are positions of the left and right interfaces of the free layer.

Second harmonic measurement



Second harmonic voltage

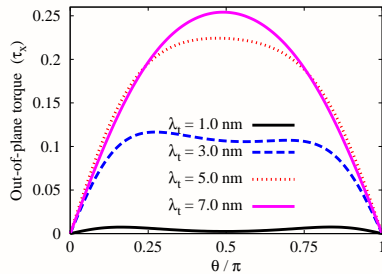
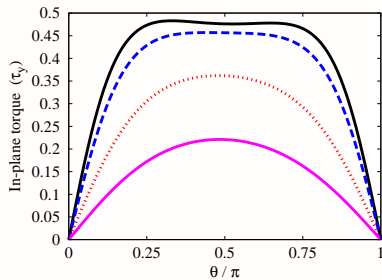
$$U_{2\omega} = -\frac{I_0^2}{4} \frac{\partial R}{\partial V} |\phi_0 \sin \phi_0| \chi_\phi$$

where $\chi_\phi(\phi_0, \Theta) = \frac{|\gamma_g|}{M_s d} \frac{b(\phi_0)}{\omega_2(\phi_0, \Theta)} \sin \phi_0$ is the **susceptibility**

Numerical results

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

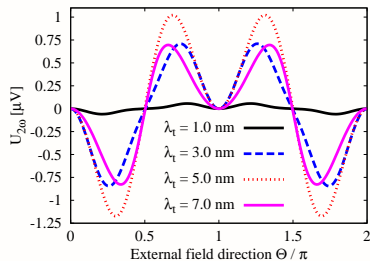
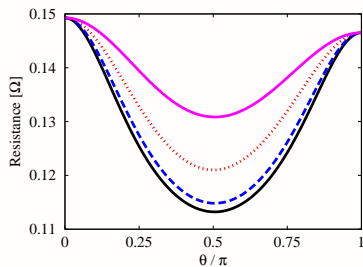
Spin transfer torque



Numerical results

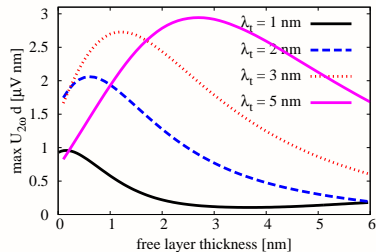
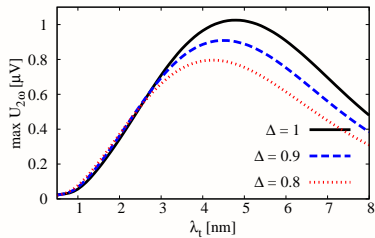
Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

Magnetoresistance and second harmonic voltage



Numerical results

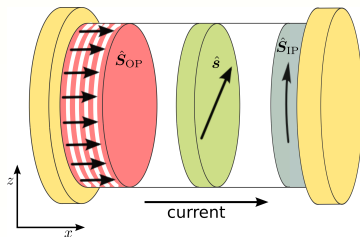
Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu



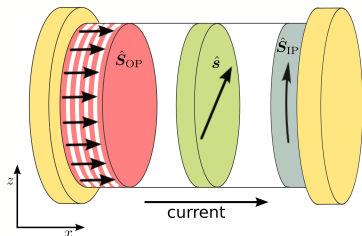
Outline

- 1 Introduction
- 2 Spin-torque in metallic spin valves
- 3 Dual spin valve
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- 5 Transverse spin transport
- 6 Transport through an perpendicular polarizer**

Modelling of transport through the perpendicular polarizer



Modelling of transport through the perpendicular polarizer



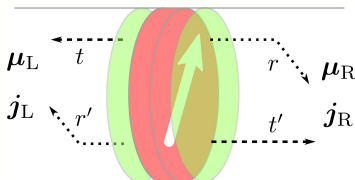
- We considered the perpendicular polarizer as a **magnetized ballistic scatterer** corresponding to a single **single interface** in our formalism.
- To calculate the transport properties of the polarizer we used **Ab initio wave function matching method**



M. Zwierzycki *et al*

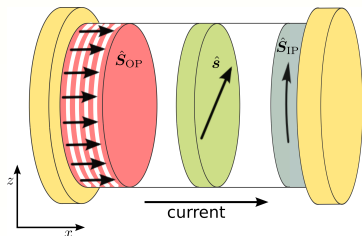
Calculating scattering matrices by wave function matching

Phys. Stat. Sol. (b) **245**, 623 – 640 (2008)



- G_{\uparrow} majority spin conductance
- G_{\downarrow} minority spin conductance
- G_{Sh} Sharvin conductance
- $G_{\uparrow\downarrow} = g_r^{\uparrow\downarrow} + i g_i^{\uparrow\downarrow}$
mixing conductance
- $T_{\uparrow\downarrow} = t_r^{\uparrow\downarrow} + i t_i^{\uparrow\downarrow}$
mixing transmission

Modelling of transport through the perpendicular polarizer



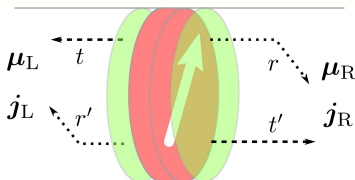
- We considered the perpendicular polarizer as a **magnetized ballistic scatterer** corresponding to a single **single interface** in our formalism.
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M. Zwierzycki *et al*

Calculating scattering matrices by wave function matching

Phys. Stat. Sol. (b) **245**, 623 – 640 (2008)



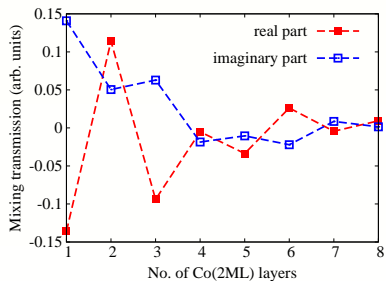
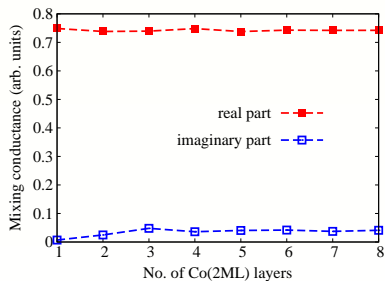
Mixing conductance

$$G_{\sigma\sigma'} = \sum_{nn'} \left[\delta_{nn'} - r_{nn'}^{\sigma} \left(r_{nn'}^{\sigma'} \right)^* \right]$$

Mixing transmission

$$T_{\sigma\sigma'} = \sum_{nn'} t_{nn'}^{\sigma} \left(t_{nn'}^{\sigma'} \right)^*$$

Numerical calculation of mixing conductance and transmission



Shep correction due to spin diffusion

Longitudinal components

$$\frac{1}{\tilde{G}_{\uparrow}} = \frac{1}{G_{\uparrow}} - \frac{1}{G_{\text{Sh}}}$$

$$\frac{1}{\tilde{G}_{\downarrow}} = \frac{1}{G_{\downarrow}} - \frac{1}{G_{\text{Sh}}}$$

Transversal componets

$$\frac{1}{\tilde{G}_{\uparrow\downarrow}} = \frac{1}{G_{\uparrow\downarrow} + T_{\uparrow\downarrow}^2 / (2G_{\text{Sh}} - G_{\uparrow\downarrow})} - \frac{1}{2G_{\text{Sh}}}$$

$$\frac{1}{\tilde{T}_{\uparrow\downarrow}} = \frac{(2G_{\text{Sh}} - G_{\uparrow\downarrow})^2 / T_{\uparrow\downarrow} - T_{\uparrow\downarrow}}{4G_{\text{Sh}}^2}.$$

Boundary conditions

Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

Boundary conditions

Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 \mathbf{j}_{\perp R} = -2 \left\{ \left(g_r^{\uparrow\downarrow} \mu_x^R - g_i^{\uparrow\downarrow} \mu_y^R - t_r^{\uparrow\downarrow} \mu_x^L + t_i^{\uparrow\downarrow} \mu_y^L \right) \hat{\mathbf{e}}_x + \left(g_r^{\uparrow\downarrow} \mu_y^R + g_i^{\uparrow\downarrow} \mu_x^R - t_r^{\uparrow\downarrow} \mu_y^L - t_i^{\uparrow\downarrow} \mu_x^L \right) \hat{\mathbf{e}}_y \right\}$$

$$e^2 \mathbf{j}_{\perp L} = -2 \left\{ \left(g_r^{\uparrow\downarrow} \mu_x^L - g_i^{\uparrow\downarrow} \mu_y^L - t_r^{\uparrow\downarrow} \mu_x^R + t_i^{\uparrow\downarrow} \mu_y^R \right) \hat{\mathbf{e}}_x + \left(g_r^{\uparrow\downarrow} \mu_y^L + g_i^{\uparrow\downarrow} \mu_x^L - t_r^{\uparrow\downarrow} \mu_y^R - t_i^{\uparrow\downarrow} \mu_x^R \right) \hat{\mathbf{e}}_y \right\}$$



Y. Tserkovnyak *et al*

Nonlocal magnetization dynamics in ferromagnetic heterostructures

Rev. Mod. Phys. **77**, 1375 (2005)

Boundary conditions

Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

Transverse components

$$e^2 j_{Rx} = -2g_r^{\uparrow\downarrow} \mu_x^R + 2g_i^{\uparrow\downarrow} \mu_y^R + 2t_r^{\uparrow\downarrow} \mu_x^L - 2t_i^{\uparrow\downarrow} \mu_y^L$$

$$e^2 j_{Ry} = -2g_r^{\uparrow\downarrow} \mu_y^R - 2g_i^{\uparrow\downarrow} \mu_x^R + 2t_r^{\uparrow\downarrow} \mu_y^L + 2t_i^{\uparrow\downarrow} \mu_x^L$$

$$e^2 j_{Lx} = -2g_r^{\uparrow\downarrow} \mu_x^L + 2g_i^{\uparrow\downarrow} \mu_y^L + 2t_r^{\uparrow\downarrow} \mu_x^R - 2t_i^{\uparrow\downarrow} \mu_y^R$$

$$e^2 j_{Ly} = -2g_r^{\uparrow\downarrow} \mu_y^L - 2g_i^{\uparrow\downarrow} \mu_x^L + 2t_r^{\uparrow\downarrow} \mu_y^R + 2t_i^{\uparrow\downarrow} \mu_x^R$$

Spin transfer torque

Spin torque acting on the free layer

$$\tau_{\parallel} = I\hat{s} \times [\hat{\mathbf{S}}_{\text{OP}} \times (a_{\text{OP}}\hat{\mathbf{S}}_{\text{OP}} + a_{\text{IP}}\hat{\mathbf{S}}_{\text{IP}})]$$

$$\tau_{\perp} = I\hat{s} \times (b_{\text{OP}}\hat{\mathbf{S}}_{\text{OP}} + b_{\text{IP}}\hat{\mathbf{S}}_{\text{IP}})$$

$$a_{\text{OP}} = -\frac{\hbar j'_{1y}}{2} \frac{|N_1/F_1|}{I \sin \theta_{\text{OP}}}$$

$$b_{\text{OP}} = \frac{\hbar j'_{1x}}{2} \frac{|N_1/F_1|}{I \sin \theta_{\text{OP}}}$$

$$a_{\text{IP}} = -\frac{\hbar j''_{2y}}{2} \frac{|F_1/N_2|}{I \sin \theta_{\text{IP}}}$$

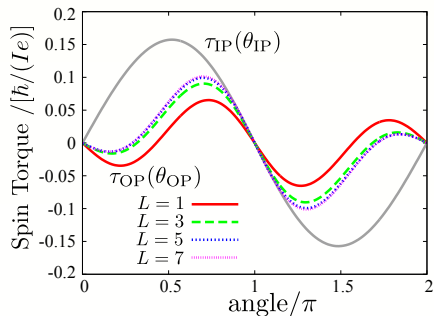
$$b_{\text{IP}} = \frac{\hbar j''_{2x}}{2} \frac{|F_1/N_2|}{I \sin \theta_{\text{IP}}}$$

where $\cos \theta_{\text{OP}} = \hat{s} \cdot \hat{\mathbf{S}}_{\text{OP}}$ and $\cos \theta_{\text{IP}} = \hat{s} \cdot \hat{\mathbf{S}}_{\text{IP}}$

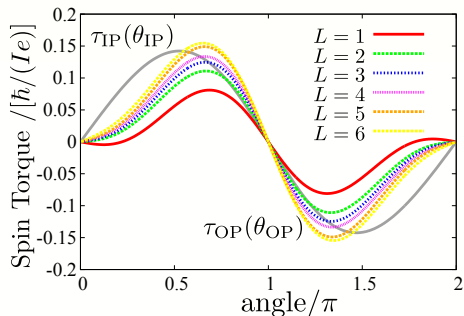
Numerical results: spin transfer torque

Cu-OPP/Cu(6)/Py(5)/Cu(8)/Py(12)-Cu

Co(2ML) / [Cu(2ML) / Co(2ML)]_L



Pt(6ML) / [Co(2ML) / Pt(3ML)]_L / Co(3ML)

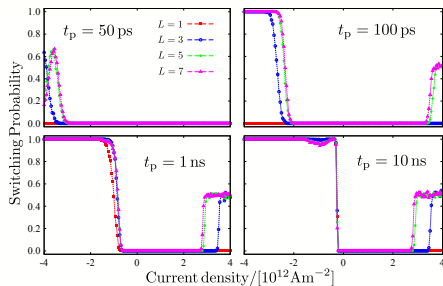


Numerical results: switching probability

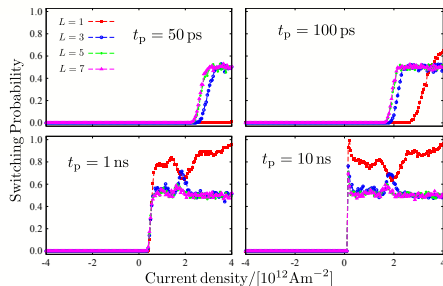
Cu-OPP/Cu(6)/Py(5)/Cu(8)/Py(12)-Cu

- **Polarizer** Co(2ML) / [Cu(2ML) / Co(2ML)]_L
- **Temperature** T=300 K

from P to AP



from AP to P



Summary

Noncollinear diffusive model is an useful and flexible treatment of spin dependent **electronic transport in metallic multilayers**, which is able to account for

- **spin transfer torque** and **magnetoresistance** in single and dual spin valves
- **nonlinear effects** in magnetoresistance
- spin transfer torque due to a **perpendicular polarizer**

Thank you for your attention

