

# Spin-dependent transport and current-induced dynamics in metallic spin valves

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60-179 Poznań

*University of California, March 2013*

# Nanoscale spin torque devices for spin electronics

Joint research project under the framework of Polish-Swiss research programme



[nanospin.agh.edu.pl](http://nanospin.agh.edu.pl)



## Partners

- AGH University of Science and Technology in Kraków  
T. Stobiecki – coordinator
- Institute of Molecular Physics in Poznań, Polish Academy of Sciences  
J. Dubowik – experiment, J. Barnaś – theory
- Ecolé Polytechnique Fédérale in Lausanne  
J.-Ph. Ansermet

## Objectives

jointly developing novel **nanoscale spintronic devices** based on the **spin transfer torque effect**, which promises unrivaled future scaling, flexibility and low power consumption



[www.zfmezo.amu.edu.pl](http://www.zfmezo.amu.edu.pl)

**Head:** prof. Józef Barnaś

## Research activities

- Single Molecular Magnets and Adatoms
- Kondo effect in quantum Dots
- Quantum and Spin Hall Effect
- Electronic transport in graphene
- Current-induced dynamics in magnetic spin valves
- Current-induced domain wall motion

## Recent projects

- European Marie Currie Research Training Network **SPINSWITCH**
- National Centre for Magnetic Nanostructures **SpinLab**
- Polish-Swiss Research Project **NanoSpin**

*with Polish Academy of Sciences, University of Technology Kraków, and École Polytechnique Fédérale de Lausanne*

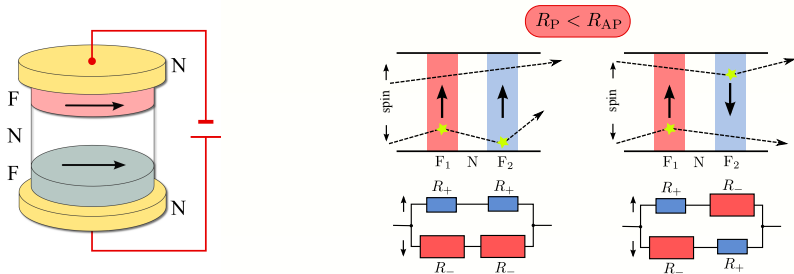


- 1 Introduction
- 2 Spin-torque in metallic spin valves
- 3 Dual spin valve
- 4 Nonlinear magnetoresistance in dual spin valves
- 5 Transverse spin transport
- 6 Transport through an perpendicular polarizer

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# Spin valves and Giant magnetoresistance



M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas  
*Giant Magnetoresistance of (001)Fe/(001)Cr Magnetic Superlattices*  
 Phys. Rev. Lett. **61**, 2472–2475 (1988)



G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn  
*Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange*  
 Phys. Rev. B **39**, 4828–4830 (1989)



R. E. Camley and J. Barnaś  
*Theory of giant magnetoresistance effects in magnetic layered structures with antiferromagnetic coupling*  
 Phys. Rev. Lett. **63**, 664–667 (1989)



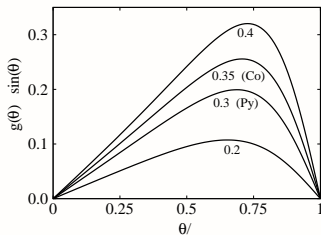
T. Valet and A. Fert  
*Theory of the perpendicular magnetoresistance in magnetic multilayers*  
 Phys. Rev. B **48**, 7099–7113 (1993)

# Current-induced dynamics and magnetization switching

Magnetization can be **switched by electric current** without need of magnetic field

## Slonczewski's model (ballistic)

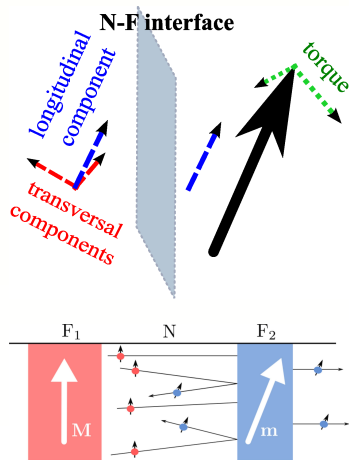
J. Magn. Magn. Mater. **159**, L1-L7 (1996)



$$\tau_{\text{Sloncz}} = \frac{I g(\theta)}{e} \hat{s} \times (\hat{s} \times \hat{S})$$

where

$$g(\theta) = \left[ -4 + (1 + P)^3 \frac{3 + \cos \theta}{4P^{3/2}} \right]^{-1}$$

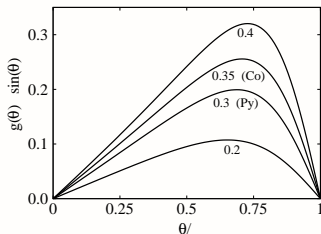


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J. Magn. Magn. Mater. **159**, L1-L7 (1996)



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## Unified description (diffusive)

Description of **spin-transfer torque** should be **consistent with** description of **giant magnetoresistance** (Valet-Fert model)



J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev  
From giant magnetoresistance to current-induced switching by spin transfer  
Phys. Rev. B **72**, 024426 (2005)

## Spin-transfer torque

$$\tau_{\theta} = a(\theta) I \hat{s} \times (\hat{s} \times \hat{S})$$

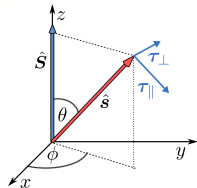
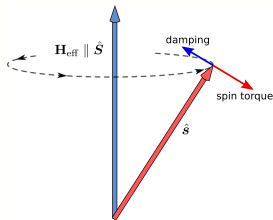
$$\tau_{\phi} = b(\theta) I \hat{s} \times \hat{S}$$



## Equation of motion

## Landau-Lifshitz-Gilbert equation

$$\frac{d\hat{\mathbf{s}}}{dt} = -|\gamma_g|\mu_0 \hat{\mathbf{s}} \times \mathbf{H}_{\text{eff}} - \alpha \hat{\mathbf{s}} \times \frac{d\hat{\mathbf{s}}}{dt} + \frac{|\gamma_g|}{M_s d} (\tau_\theta + \tau_\phi)$$



## Effective magnetic field

$$\mathbf{H}_{\text{eff}} = -H_{\text{ext}}\hat{\mathbf{e}}_z - H_{\text{ani}}(\hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_z)\hat{\mathbf{e}}_z + \mathbf{H}_{\text{demag}}$$

## Spin-transfer torque

$$\tau_\theta = a(\theta) I \hat{\mathbf{s}} \times (\hat{\mathbf{s}} \times \hat{\mathbf{S}})$$

$$\tau_\phi = b(\theta) I \hat{\mathbf{s}} \times \hat{\mathbf{S}}$$

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# Mathematical description

## Two channels model

bulk resistivities

$$\rho_{\uparrow(\downarrow)} = 2\rho^* (1 \mp \beta)$$

interface resistances

$$R_{\uparrow(\downarrow)} = 2R^* (1 \mp \gamma)$$

- $\beta$  bulk asymmetry parameters
- $\gamma$  interfacial asymmetry parameter

## Diffusive transport

$$\frac{\partial^2 (\bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow})}{\partial x^2} = \frac{1}{l_{sf}^2} (\bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow})$$

$$\frac{\partial^2 (\bar{\mu}_{\uparrow} + \bar{\mu}_{\downarrow})}{\partial x^2} = \eta \frac{\partial^2 (\bar{\mu}_{\uparrow} - \bar{\mu}_{\downarrow})}{\partial x^2}$$



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for electrochemical potentials we get

$$\bar{\mu}_{\uparrow} = (1 + \eta) \left[ Ae^{x/l_{sf}} + Be^{-x/l_{sf}} \right] + Cx + G$$

$$\bar{\mu}_{\downarrow} = (\eta - 1) \left[ Ae^{x/l_{sf}} + Be^{-x/l_{sf}} \right] + Cx + G$$



J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev  
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# Mathematical description

## Magnetic layer

$$\check{\mu} = \bar{\mu}_0 \check{1} + g \check{\sigma}_z$$

$$\bar{\mu}_0 = \frac{\bar{\mu}_\uparrow + \bar{\mu}_\downarrow}{2}, \quad g = \frac{\bar{\mu}_\uparrow - \bar{\mu}_\downarrow}{2}$$

with  $g$  being **spin accumulation**

$$\check{j} = -\rho(E_F) \check{D} \frac{\partial \check{\mu}}{\partial x}$$

$$\check{j} = \frac{1}{2} (j_0 \check{1} + j_z \check{\sigma}_z)$$

with  $j_z$  being **spin current**

## Diffusive transport

$$\frac{\partial^2 (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)}{\partial x^2} = \frac{1}{l_{sf}^2} (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)$$

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J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev

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Phys. Rev. B **72**, 024426 (2005)

# Mathematical description

## Nonmagnetic layer

has no natural quantization axis

$$\begin{aligned}\check{\mu} &= \bar{\mu}_0 \check{\mathbf{I}} + \mathbf{g} \cdot \check{\boldsymbol{\sigma}} \\ \check{\mathbf{j}} &= \frac{1}{2} (j_0 \check{\mathbf{I}} + \mathbf{j} \cdot \check{\boldsymbol{\sigma}})\end{aligned}$$

where

$$\mathbf{g} = (g_x, g_y, g_z), \quad \mathbf{j} = (j_x, j_y, j_z)$$

are 3D vectors written in the coordinate system of one of the adjacent magnetic layers

## Diffusive transport

$$\begin{aligned}\frac{\partial^2 (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)}{\partial x^2} &= \frac{1}{l_{\text{sf}}^2} (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow) \\ \frac{\partial^2 (\bar{\mu}_\uparrow + \bar{\mu}_\downarrow)}{\partial x^2} &= \eta \frac{\partial^2 (\bar{\mu}_\uparrow - \bar{\mu}_\downarrow)}{\partial x^2}\end{aligned}$$

for electrochemical potentials we get

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Phys. Rev. B **72**, 024426 (2005)

## Boundary conditions at N/F interface

- **particle current** is continuous across all interfaces

$$e^2 j_0 = (G_{\uparrow} + G_{\downarrow})(\bar{\mu}_0^F - \bar{\mu}_0^N) + (G_{\uparrow} - G_{\downarrow})(g_z^F - g_z^N)$$

- **spin current component parallel to the magnetization** is continuous across the interface

$$e^2 j_z = (G_{\uparrow} - G_{\downarrow})(\bar{\mu}_0^F - \bar{\mu}_0^N) + (G_{\uparrow} + G_{\downarrow})(g_z^F - g_z^N)$$

- **transversal component of the spin current** vanishes in the magnetic layer – jump at the interface

$$e^2 j_x = -2\text{Re}\{G_{\uparrow\downarrow}\}g_x^N + 2\text{Im}\{G_{\uparrow\downarrow}\}g_y^N$$

$$e^2 j_y = -2\text{Re}\{G_{\uparrow\downarrow}\}g_y^N - 2\text{Im}\{G_{\uparrow\downarrow}\}g_x^N$$



A. Brataas, Yu.V. Nazarov, G.E.W. Bauer

*Spin-transport in multi-terminal normal metal-ferromagnet systems with non-collinear magnetizations*  
 Eur. Phys. J. B **22**, 99 (2001)

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$$e^2 j_y = -2\text{Re}\{G_{\uparrow\downarrow}\}g_y^N - 2\text{Im}\{G_{\uparrow\downarrow}\}g_x^N$$

## Spin-transfer torque

$$\tau = \frac{\hbar}{2}(\mathbf{j}_{\perp L} - \mathbf{j}_{\perp R})$$



## Boundary conditions at N/F interface

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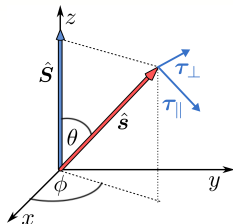
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$$e^2 j_y = -2\text{Re}\{G_{\uparrow\downarrow}\}g_y^N - 2\text{Im}\{G_{\uparrow\downarrow}\}g_x^N$$



## Components

in-plane

$$\tau_{\theta} = -\frac{\hbar}{2} j_y' |_{N/F}$$

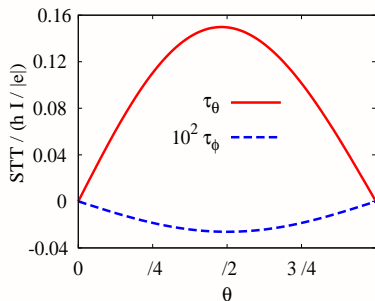
out-of-plane

$$\tau_{\phi} = \frac{\hbar}{2} j_x' |_{N/F}$$

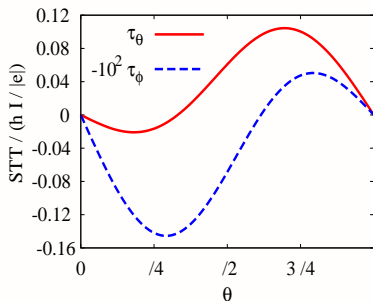
# Results

Calculations for real structures

## Standard spin valve Py(20)/Cu(10)/Py(8)



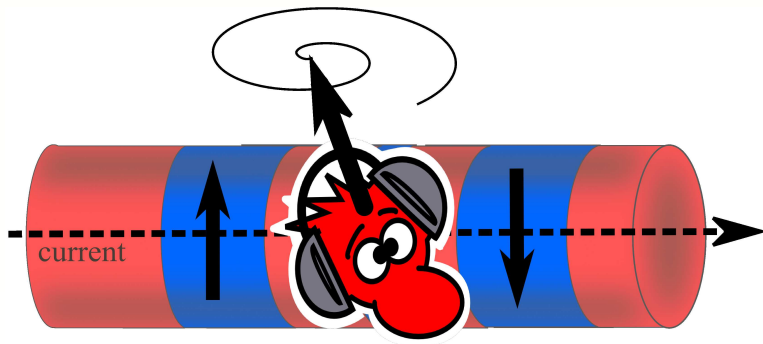
## Nonstandard spin valve Co(8)/Cu(10)/Py(8)



# Outline

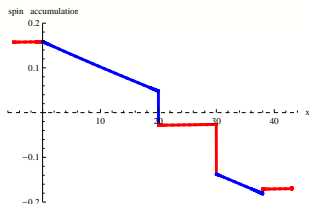
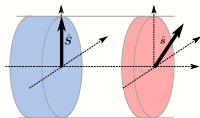
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# What is a dual spin valve?

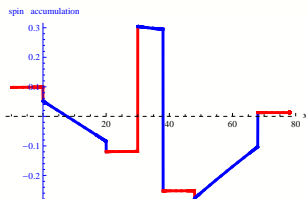
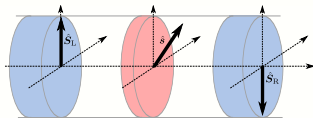


## Spin accumulation in dual spin valve

spin accumulation in single spin valve



spin accumulation in dual spin valve

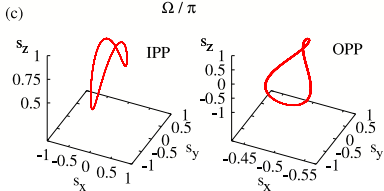
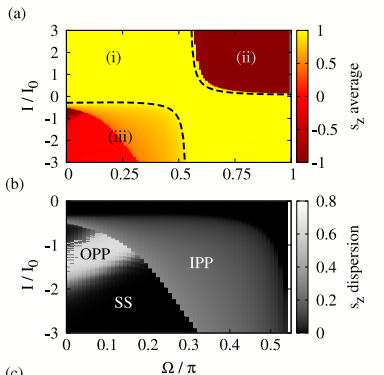
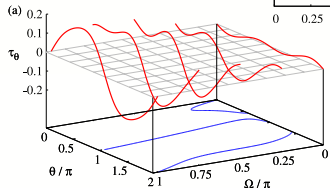
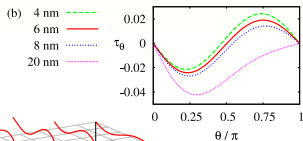
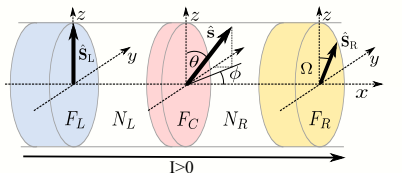


L. Berger

*Multilayer Configuration for Experiments of Spin Precession Induced by a DC Current*

*J. Appl. Phys.* **93**, 7683 (2003)

## Noncollinear configurations in dual spin valves



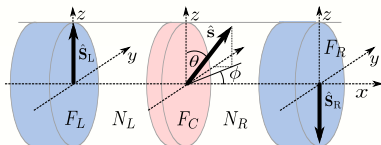
P.B., M. Gmitra, J. Barnaś

Phys. Rev. B **80**, 174404 (2009)

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# Nonlinear magnetoresistance in dual spin valves



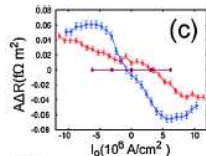
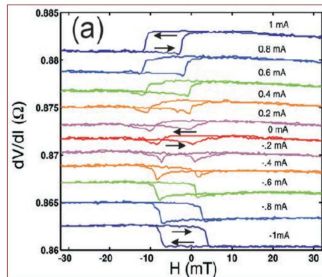
## Experimental works



A. Aziz, O. P. Wessely, M. Ali, D. M. Edwards, C. H. Marrows, B. J. Hickey, and M. G. Blamire  
*Nonlinear giant magnetoresistance in dual spin valves*  
*Phys. Rev. Lett.* **103**, 237203 (2009)



N. Banerjee, A. Aziz, M. Ali, J. W. A. Robinson, B. J. Hickey, and M. G. Blamire  
*Thickness dependence and the role of spin transfer torque in nonlinear giant magnetoresistance of permalloy dual spin valves*  
*Phys. Rev. B* **82**, 224402 (2010)



Experimental results [Aziz *et al.*, PRL (2009)]: (b) minor loops for  $\text{Co}_{90}\text{Fe}_{10}(6)/\text{Cu}(4)/\text{Py}(1)/\text{Cu}(2)/\text{Co}_{90}\text{Fe}_{10}(6)/\text{IrMn}(10)$  (c) Py thickness: 1 nm (blue), 2 nm (red), and 8 nm (magenta)



# Nonlinear magnetoresistance in dual spin valves

## Model's assumptions

- Spin accumulation in the central layer changes density of states on Fermi level
- This may change bulk/interfacial material parameters in the central layer

We extended the diffusion transport model



J. Barnaś, A. Fert, M. Gmitra, I. Weymann, V.K. Dugaev

*From giant magnetoresistance to current-induced switching by spin transfer*  
 Phys. Rev. B **72**, 024426 (2005)

### Bulk contribution

$$\rho^* = \rho_0^* + q \langle g \rangle$$

$$\beta = \beta_0 + \xi \langle g \rangle$$

### Interfacial contribution

$$R^* = R_0^* + q' g(x_i)$$

$$\gamma = \gamma_0 + \xi' g(x_i)$$

where

- $g(x)$  is spin accumulation
- $\rho_0^*$  and  $R_0^*$  are *zero-current* bulk resistivity and interfacial resistance
- $\beta_0$  and  $\gamma_0$  are *zero-current* bulk/interfacial asymmetry parameters
- $q, \xi, q', \xi'$  are phenomenological parameters

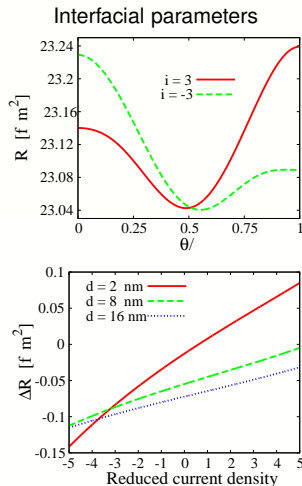
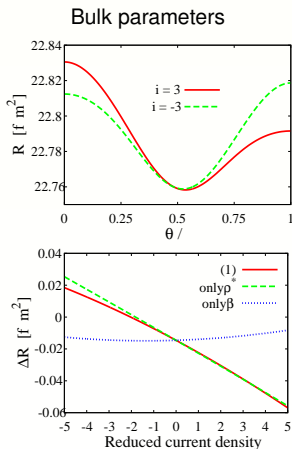


P. Baláž, and J. Barnaś

*Nonlinear magnetotransport in dual spin valves*  
 Phys. Rev. B **82**, 104430 (2010)

# Nonlinear magnetoresistance in dual spin vales

Calculations for Cu - Co(6) / Cu(4) / Py(2) / Cu(2) / Co(6) / IrMn(10) - Cu



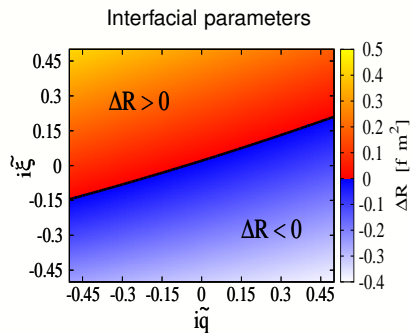
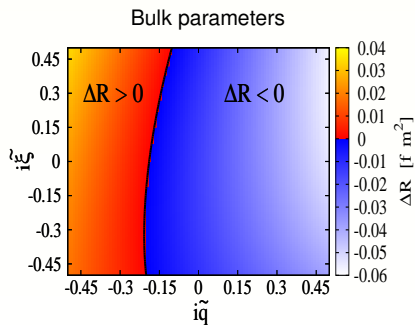
P. Baláz, and J. Barnaś

*Nonlinear magnetotransport in dual spin vales*  
 Phys. Rev. B **82**, 104430 (2010)

for  $\tilde{q} = 0.1$ ,  $\tilde{\xi} = 0.1$ ,  $I_0 = 10^8 \text{ Acm}^{-2}$

# Nonlinear magnetoresistance in dual spin valves

Calculations for Cu - Co(6) / Cu(4) / Py(2) / Cu(2) / Co(6) / IrMn(10) - Cu



for **interfaces**  $\Delta R$  is **symmetric** with current density



P. Baláz, and J. Barnaś

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# Transverse spin diffusion

## Motivation



T. Taniguchi, H. Imamura

*Spin pumping in ferromagnetic multilayers*  
 Mod. Phys. Lett. B **22**, 2909 – 2929 (2008)



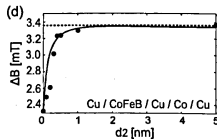
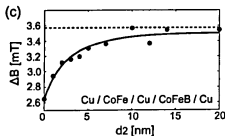
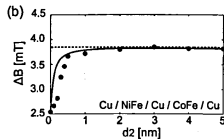
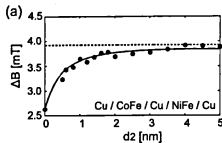
T. Taniguchi, S. Yakata, H. Imamura, Y. Ando

*Determination of Penetration Depth of Transverse Spin Current in Ferromagnetic Metals by Spin Pumping*  
 arXiv:0708.3528v3 [cond-mat.mes-hall]



T. Taniguchi, S. Yakata, H. Imamura, Y. Ando

*Penetration Depth of Transverse Spin Current in Ferromagnetic Metals*  
 IEEE Trans. Mag. **44**, 2636 (2008); arXiv:0806.3315v1 [cond-mat.mes-hall]



## Estimated penetration length

- Py 3.7 nm
- CoFe 2.5 nm
- CoFeB 12 nm
- Co 1.7 nm

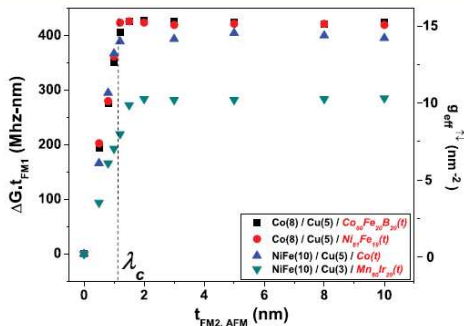
# Another experiment



A. Gosh, S. Auffret, U. Ebels, W. E. Bailey

Penetration depth of transverse current in ultrathin ferromagnets

Phys. Rev. Lett. **109**, 127202 (2012)



- $\lambda_c = 1.2 \pm 0.1$  nm
- algebraic decay with the  $F_2$  thickness

# Transverse spin diffusion

## Modified continuity equation

$$\frac{df_0}{dt} = -\frac{\partial j_0}{\partial x},$$

$$\frac{d\mathbf{f}}{dt} = \frac{J}{\hbar} (\hat{\mathbf{S}} \times \mathbf{f}) - \frac{\partial \mathbf{j}}{\partial x} + \frac{\mathbf{f}}{\tau_{\text{sf}}},$$

## Current tensor

$$\check{\mathbf{j}} = \begin{pmatrix} j_{\uparrow} & j_{\uparrow\downarrow} \\ j_{\downarrow\uparrow} & j_{\downarrow} \end{pmatrix} = - \begin{pmatrix} D_{\uparrow} \frac{\partial f_{\uparrow}}{\partial x} & D_{\perp} \frac{\partial f_{\uparrow\downarrow}}{\partial x} \\ D_{\perp} \frac{\partial f_{\downarrow\uparrow}}{\partial x} & D_{\downarrow} \frac{\partial f_{\downarrow}}{\partial x} \end{pmatrix}$$

where

$$D_{\uparrow(\downarrow)} = D_0 / (1 \mp \beta)$$

$$D_{\perp} = \frac{D_{\uparrow} + D_{\downarrow}}{2}$$



A. Shpiro, P. Lévy, and S. Zhang

*Self-consistent treatment of nonequilibrium spin torques in magnetic multilayers*  
 Phys. Rev. B **67** 104430 (2002)

## Charge and longitudinal components

$$\frac{\partial^2 f_z}{\partial x^2} = \frac{f_z}{\lambda_{\text{sf}}^2}$$

$$\frac{\partial^2 f_0}{\partial x^2} = -\beta \frac{\partial^2 f_z}{\partial x^2}$$

where  $\lambda_{\text{sf}} = \sqrt{D_0 \tau_{\text{sf}}}$ .

## Transverse spin components

$$\frac{\partial^2 f_x}{\partial x^2} = -\frac{f_y}{\lambda_J^2} + \frac{f_x}{\lambda_{\perp}^2}$$

$$\frac{\partial^2 f_y}{\partial x^2} = \frac{f_x}{\lambda_J^2} + \frac{f_y}{\lambda_{\perp}^2}$$

where  $\lambda_J = \sqrt{\hbar D_{\perp} / J}$  and  $\lambda_{\perp} = \sqrt{D_{\perp} \tau_{\text{sf}}}$

# Solution for the transver components

## Longitudinal spin accumulation

$$\mu_z(x) = A e^{x/\lambda_{sf}} + B e^{-x/\lambda_{sf}}$$

- $\lambda_{sf}$ : spin diffusion length

## Transverse spin accumulation

$$\mu_x(x) = C_1 e^{x/l_-} + C_2 e^{-x/l_-} + C_3 e^{x/l_+} + C_4 e^{-x/l_+}$$

$$\frac{1}{l_{\pm}} = \frac{1}{\lambda_t} \pm i \frac{1}{\lambda_p}$$

- $\lambda_t$ : transverse decay length
- $\lambda_p$ : precessional length

we define  $\Delta = \lambda_t/\lambda_p$ , which is  $\Delta < 1$

$$\frac{1}{l_{\pm}} = \frac{1 \pm i\Delta}{\lambda_t}$$



# Solution for the transverse components

## Longitudinal charge and spin current

$$\frac{1}{\rho(E_F)} j_0 = -2C \frac{D_0}{1-\beta^2}$$

$$\frac{1}{\rho(E_F)} j_z(x) = \frac{\beta}{\rho(E_F)} j_0 - \frac{2D_0}{\lambda_{sf}} \left[ A e^{x/\lambda_{sf}} - B e^{-x/\lambda_{sf}} \right]$$

## Transverse spin current

$$\frac{1}{\rho(E_F)} j_{x(y)} = \pm \frac{D_{\perp}}{\lambda_t} \left\{ \left[ (\Delta A_1 - B_1) \sin\left(\frac{\Delta x}{\lambda_t}\right) \mp (A_1 + \Delta B_1) \cos\left(\frac{\Delta x}{\lambda_t}\right) \right] e^{x/\lambda_t} \pm \right.$$

$$\left. \left[ (\Delta A_2 + B_2) \sin\left(\frac{\Delta x}{\lambda_t}\right) \pm (A_2 - \Delta B_2) \cos\left(\frac{\Delta x}{\lambda_t}\right) \right] e^{-x/\lambda_t} \right\}$$

## Boundary conditions at the N/F interfaces

## Longitudinal components

$$e^2 j_0 = (G_{\uparrow} + G_{\downarrow})(\mu_0^F - \mu_0^N) + (G_{\uparrow} - G_{\downarrow})(\mu_z^F - \mu_z^N)$$

$$e^2 j_z = (G_{\uparrow} - G_{\downarrow})(\mu_0^F - \mu_0^N) + (G_{\uparrow} + G_{\downarrow})(\mu_z^F - \mu_z^N)$$

## Transversal components

$$e^2 j_x = 2 \operatorname{Re} \{ G_{\uparrow\downarrow} \} (\mu_x^F - \mu_x^N) + 2 \operatorname{Im} \{ G_{\uparrow\downarrow} \} (\mu_y^F - \mu_y^N)$$

$$e^2 j_y = 2 \operatorname{Re} \{ G_{\uparrow\downarrow} \} (\mu_y^F - \mu_y^N) - 2 \operatorname{Im} \{ G_{\uparrow\downarrow} \} (\mu_x^F - \mu_x^N)$$

## Current continuity

$$j_x^F = j_x^N = j_x$$

$$j_y^F = j_y^N = j_y$$

$$j_z^F = j_z^N = j_z$$

## Spin transfer torque

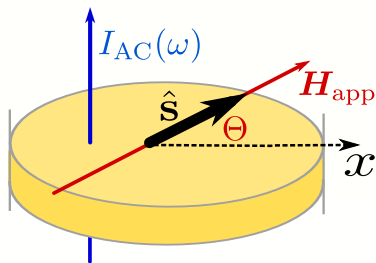
## Spin transfer torque

$$\tau_x = \frac{\hbar}{2} \int_{x_L}^{x_R} \frac{\partial j_x}{\partial x} dx = \frac{\hbar}{2} [j_x(x_R) - j_x(x_L)] \equiv \tau_{\perp}$$

$$\tau_y = \frac{\hbar}{2} \int_{x_L}^{x_R} \frac{\partial j_y}{\partial x} dx = \frac{\hbar}{2} [j_y(x_R) - j_y(x_L)] \equiv \tau_{\parallel}$$

where  $x_L$  and  $x_R$  are positions of the left and right interfaces of the free layer.

## Second harmonic measurement



## Second harmonic voltage

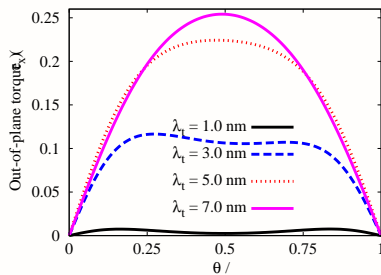
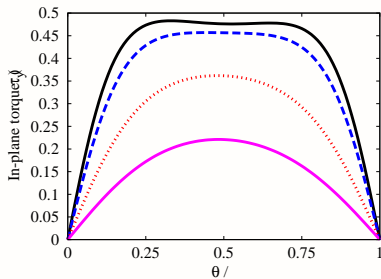
$$U_{2\omega} = -\frac{I_0^2}{4} \frac{\partial R}{\partial V} |\phi_0 \sin \phi_0| \chi_\phi$$

where  $\chi_\phi(\phi_0, \Theta) = \frac{|\gamma_g|}{M_s d} \frac{b(\phi_0)}{\omega_2(\phi_0, \Theta)} \sin \phi_0$  is the **susceptibility**

# Numerical results

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

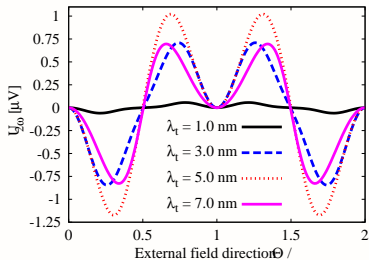
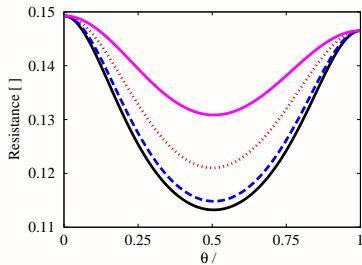
## Spin transfer torque



# Numerical results

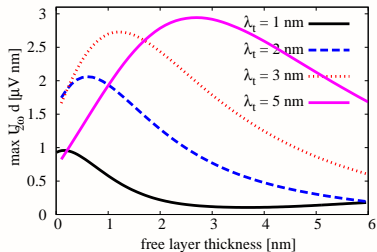
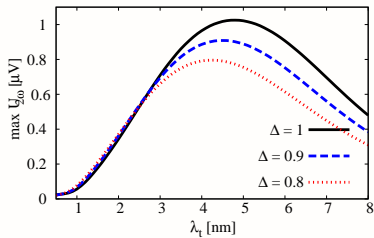
Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

## Magnetoresistance and second harmonic voltage



# Numerical results

Cu – CoFe(2.5) / Cu(2.8) / CoFe(2.5) / Cu(2.8) / CoFe(3.5) – Cu

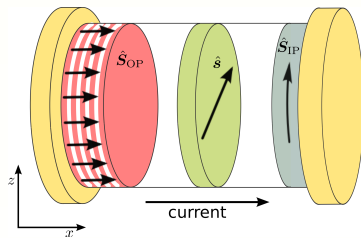


# Outline

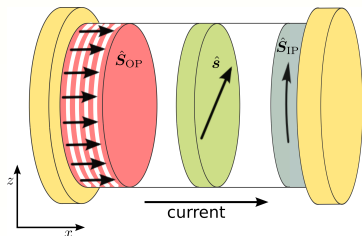
- 1 Introduction
- 2 Spin-torque in metallic spin valves
- 3 Dual spin valve
- 4 Nonlinear magnetoresistance in dual spin valves
- 5 Transverse spin transport
- 6 Transport through an perpendicular polarizer**



## Modelling of transport through the perpendicular polarizer



## Modelling of transport through the perpendicular polarizer



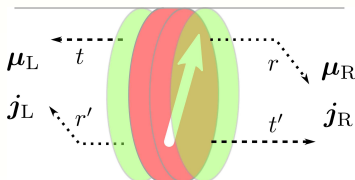
- We considered the perpendicular polarizer as a **magnetized ballistic scatterer** corresponding to a single **single interface** in our formalism.
- To calculate the transport properties of the polarizer we used **Ab initio wave function matching method**



M. Zwierzycki *et al*

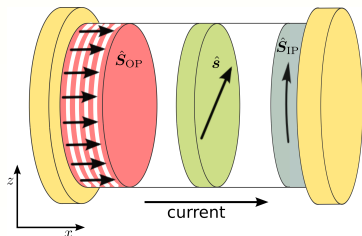
Calculating scattering matrices by wave function matching

Phys. Stat. Sol. (b) **245**, 623 – 640 (2008)



- $G_{\uparrow}$  majority spin conductance
- $G_{\downarrow}$  minority spin conductance
- $G_{\text{Sh}}$  Sharvin conductance
- $G_{\uparrow\downarrow} = g_r^{\uparrow\downarrow} + i g_i^{\uparrow\downarrow}$   
mixing conductance
- $T_{\uparrow\downarrow} = t_r^{\uparrow\downarrow} + i t_i^{\uparrow\downarrow}$   
mixing transmission

## Modelling of transport through the perpendicular polarizer



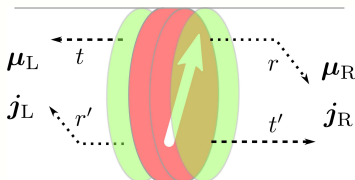
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M. Zwierzycki *et al*

Calculating scattering matrices by wave function matching

Phys. Stat. Sol. (b) **245**, 623 – 640 (2008)



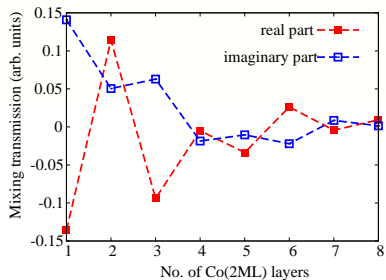
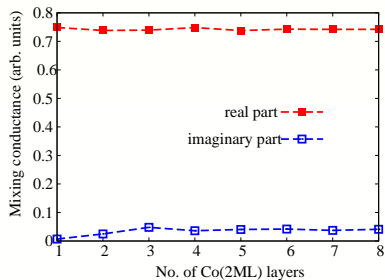
### Mixing conductance

$$G_{\sigma\sigma'} = \sum_{nn'} \left[ \delta_{nn'} - r_{nn'}^{\sigma} \left( r_{nn'}^{\sigma'} \right)^* \right]$$

### Mixing transmission

$$T_{\sigma\sigma'} = \sum_{nn'} t_{nn'}^{\sigma} \left( t_{nn'}^{\sigma'} \right)^*$$

# Numerical calculation of mixing conductance and transmission



# Shep correction due to spin diffusion

## Longitudinal components

$$\frac{1}{\tilde{G}_{\uparrow}} = \frac{1}{G_{\uparrow}} - \frac{1}{G_{\text{Sh}}}$$

$$\frac{1}{\tilde{G}_{\downarrow}} = \frac{1}{G_{\downarrow}} - \frac{1}{G_{\text{Sh}}}$$

## Transversal componets

$$\frac{1}{\tilde{G}_{\uparrow\downarrow}} = \frac{1}{G_{\uparrow\downarrow} + T_{\uparrow\downarrow}^2 / (2G_{\text{Sh}} - G_{\uparrow\downarrow})} - \frac{1}{2G_{\text{Sh}}}$$

$$\frac{1}{\tilde{T}_{\uparrow\downarrow}} = \frac{(2G_{\text{Sh}} - G_{\uparrow\downarrow})^2 / T_{\uparrow\downarrow} - T_{\uparrow\downarrow}}{4G_{\text{Sh}}^2}.$$

# Boundary conditions

## Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

## Boundary conditions

## Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 \mathbf{j}_{\perp R} = -2 \left\{ \left( g_r^{\uparrow\downarrow} \mu_x^R - g_i^{\uparrow\downarrow} \mu_y^R - t_r^{\prime\uparrow\downarrow} \mu_x^L + t_i^{\prime\uparrow\downarrow} \mu_y^L \right) \hat{\mathbf{e}}_x + \left( g_r^{\uparrow\downarrow} \mu_y^R + g_i^{\uparrow\downarrow} \mu_x^R - t_r^{\prime\uparrow\downarrow} \mu_y^L - t_i^{\prime\uparrow\downarrow} \mu_x^L \right) \hat{\mathbf{e}}_y \right\}$$

$$e^2 \mathbf{j}_{\perp L} = -2 \left\{ \left( g_r^{\prime\uparrow\downarrow} \mu_x^L - g_i^{\prime\uparrow\downarrow} \mu_y^L - t_r^{\uparrow\downarrow} \mu_x^R + t_i^{\uparrow\downarrow} \mu_y^R \right) \hat{\mathbf{e}}_x + \left( g_r^{\prime\uparrow\downarrow} \mu_y^L + g_i^{\prime\uparrow\downarrow} \mu_x^L - t_r^{\uparrow\downarrow} \mu_y^R - t_i^{\uparrow\downarrow} \mu_x^R \right) \hat{\mathbf{e}}_y \right\}$$



Y. Tserkovnyak *et al*

*Nonlocal magnetization dynamics in ferromagnetic heterostructures*

*Rev. Mod. Phys.* **77**, 1375 (2005)

# Boundary conditions

## Longitudinal components

$$e^2 j_0 = (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

$$e^2 j_z = (\tilde{G}_\uparrow - \tilde{G}_\downarrow)(\mu_0^R - \mu_0^L) + (\tilde{G}_\uparrow + \tilde{G}_\downarrow)(\mu_z^R - \mu_z^L)$$

## Transverse components

$$e^2 j_{Rx} = -2g_r^{\uparrow\downarrow} \mu_x^R + 2g_i^{\uparrow\downarrow} \mu_y^R + 2t_r^{\uparrow\downarrow} \mu_x^L - 2t_i^{\uparrow\downarrow} \mu_y^L$$

$$e^2 j_{Ry} = -2g_r^{\uparrow\downarrow} \mu_y^R - 2g_i^{\uparrow\downarrow} \mu_x^R + 2t_r^{\uparrow\downarrow} \mu_y^L + 2t_i^{\uparrow\downarrow} \mu_x^L$$

$$e^2 j_{Lx} = -2g_r^{\uparrow\downarrow} \mu_x^L + 2g_i^{\uparrow\downarrow} \mu_y^L + 2t_r^{\uparrow\downarrow} \mu_x^R - 2t_i^{\uparrow\downarrow} \mu_y^R$$

$$e^2 j_{Ly} = -2g_r^{\uparrow\downarrow} \mu_y^L - 2g_i^{\uparrow\downarrow} \mu_x^L + 2t_r^{\uparrow\downarrow} \mu_y^R + 2t_i^{\uparrow\downarrow} \mu_x^R$$



## Spin transfer torque

## Spin torque acting on the free layer

$$\tau_{\parallel} = I\hat{s} \times [\hat{\mathbf{S}}_{\text{OP}} \times (a_{\text{OP}}\hat{\mathbf{S}}_{\text{OP}} + a_{\text{IP}}\hat{\mathbf{S}}_{\text{IP}})]$$

$$\tau_{\perp} = I\hat{s} \times (b_{\text{OP}}\hat{\mathbf{S}}_{\text{OP}} + b_{\text{IP}}\hat{\mathbf{S}}_{\text{IP}})$$

$$a_{\text{OP}} = -\frac{\hbar j'_{1y}}{2} \frac{|N_1/F_1|}{I \sin \theta_{\text{OP}}}$$

$$b_{\text{OP}} = \frac{\hbar j'_{1x}}{2} \frac{|N_1/F_1|}{I \sin \theta_{\text{OP}}}$$

$$a_{\text{IP}} = -\frac{\hbar j''_{2y}}{2} \frac{|F_1/N_2|}{I \sin \theta_{\text{IP}}}$$

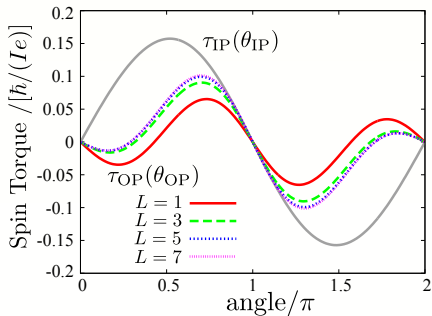
$$b_{\text{IP}} = \frac{\hbar j''_{2x}}{2} \frac{|F_1/N_2|}{I \sin \theta_{\text{IP}}}$$

where  $\cos \theta_{\text{OP}} = \hat{s} \cdot \hat{\mathbf{S}}_{\text{OP}}$  and  $\cos \theta_{\text{IP}} = \hat{s} \cdot \hat{\mathbf{S}}_{\text{IP}}$

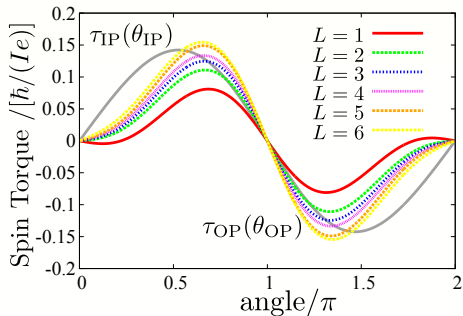
# Numerical results: spin transfer torque

Cu-OPP/Cu(6)/Py(5)/Cu(8)/Py(12)-Cu

Co(2ML) / [ Cu(2ML) / Co(2ML) ]<sub>L</sub>



Pt(6ML) / [ Co(2ML) / Pt(3ML) ]<sub>L</sub> / Co(3ML)

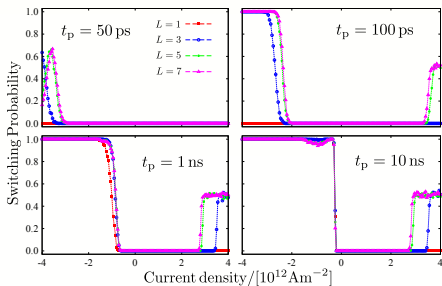


# Numerical results: switching probability

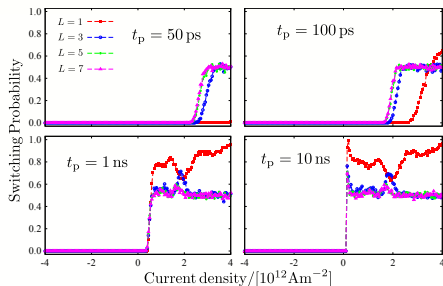
Cu-OPP/Cu(6)/Py(5)/Cu(8)/Py(12)-Cu

- **Polarizer** Co(2ML) / [ Cu(2ML) / Co(2ML) ]<sub>L</sub>
- **Temperature** T=300 K

**from P to AP**



**from AP to P**



# Summary

**Noncollinear diffusive model** is an useful and flexible treatment of spin dependent **electronic transport in metallic multilayers**, which is able to account for

- **spin transfer torque** and **magnetoresistance** in single and dual spin valves
- **nonlinear effects** in magnetoresistance
- spin transfer torque due to a **perpendicular polarizer**

# Thank you for your attention