Influence of spin pumping on spin wave spectra of single and double magnetic layers

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We present results of our analysis of the influence of spin pumping on spin waves in a **double magnetic layers of YIG**. The magnetization dynamics has been modeled with the **Landau-Lifshitz-Gilbert equation** [1], and effect of interface perpendicular magnetic anisotropy has been included. Generally, spin pumping contributes to the intrinsic Gilbert damping. In case of two magnetic layers separated by a nonmagnetic metallic spacer, the spin current pumped into the spacer leads to an additional dynamic coupling between the layers [2].

[1] M. Vohl, J. Barnaś, P. Grünberg, Phys. Rev. B 39, 12003 (1989); J. Barnaś, P. Grünberg, J. Magn. Magn. Mater. 82, 186 (1989).
[2] Y. Zhou *et al*, Phy. Rev. B 88, 184403 (2013); Y. Tserkovnyak *et al*, Rev. Mod. Phys. 77, 1375 (2005).

Bulk magnetization dynamics

The dynamics in the j-th layer shall be described by Landau-Lifshitz-Gilbert equation (LLG)

$$\frac{\mathrm{d}\boldsymbol{M}_j}{\mathrm{d}t} = -\gamma_j \mu_0 \boldsymbol{M}_j \times \boldsymbol{H}_{\mathrm{eff}\,j} + \frac{\alpha_j}{M_{\mathrm{s}j}} \boldsymbol{M}_j \times \frac{\mathrm{d}\boldsymbol{M}_j}{\mathrm{d}t}$$

where $\boldsymbol{M}_j = \boldsymbol{M}_j(\boldsymbol{r},t)$ is magnetization of the j-th layer, $M_{\mathrm{s}j} = |\boldsymbol{M}_j|$ is the saturated magnetization, α_j is the Gilbert damping parameter of j-the magnetization, μ_0 is the vacuum permeability, and $\gamma_j = |e|g_j/(2m)$ is the gyromagnetic ratio. and g_j the Landee g-factor of the j-the layer.
$\boldsymbol{H}_{\mathrm{eff}\,j} = \boldsymbol{H}_{\mathrm{eff}\,j}(\boldsymbol{r},t)$ is the effective magnetic field in the j-th magnetic layer

$$\boldsymbol{H}_{\mathrm{eff}\,j}(\boldsymbol{r},t) = H_0\,\hat{\boldsymbol{e}}_z + \boldsymbol{h}_j(\boldsymbol{r},t) + \frac{2A_j}{\mu_0 M_{\mathrm{s}j}^2}\nabla^2 \boldsymbol{M}_j(\boldsymbol{r},t)$$

where H_0 is the applied magnetic field, $\boldsymbol{h}_j(\boldsymbol{r},t)$ is the dipolar exchange field, A_j is the exchange stiffness in j-th magnetic layer.

We linearize LLG assuming $\boldsymbol{M}_j(\boldsymbol{r},t) = M_{\mathrm{s}j}\hat{\boldsymbol{e}}_z + \boldsymbol{m}_j(\boldsymbol{r},t)$, where $\boldsymbol{m}_j(\boldsymbol{r},t) = \boldsymbol{m}_j(\boldsymbol{r})\,e^{-i\omega t}$. The dipolar-exchange field is $\boldsymbol{h}_j(\boldsymbol{r},t) = \boldsymbol{h}_j(\boldsymbol{r})\,e^{-i\omega t}$ obeys the magnetostatic equations $\nabla \times \boldsymbol{h}_j(\boldsymbol{r}) = 0$, and $\nabla \cdot [\boldsymbol{h}_j(\boldsymbol{r}) + \boldsymbol{m}_j(\boldsymbol{r})] = 0$, which indicates that $\boldsymbol{h}_j(\boldsymbol{r}) = -\nabla\psi_j(\boldsymbol{r})$, where $\psi_j(\boldsymbol{r})$ is the magnetostatic scalar potential, which obeys

$$\nabla^2\psi_j(\boldsymbol{r}) - \left(\frac{\partial m_{j,x}}{\partial x} + \frac{\partial m_{j,y}}{\partial y}\right) = 0$$

Boundary conditions

The double layer has 4 interfaces.

External interfaces

On the external interfaces Rado-Weertman boundary conditions have to be fulfilled

$$2A_j\boldsymbol{M}_j \times \frac{\partial \boldsymbol{M}_j}{\partial \hat{\boldsymbol{n}}_j} - 2K^{\mathrm{s}}{}_j\,(\boldsymbol{M}_j.\hat{\boldsymbol{n}}_j)(\boldsymbol{M}_j \times \hat{\boldsymbol{n}}_j) - J_{\mathrm{s}}\,\boldsymbol{M}_j \times \hat{\boldsymbol{e}}_z \times \boldsymbol{M}_j + \frac{\hbar\, g_{\mathrm{r}j}}{4\pi}\boldsymbol{M}_1 \times \frac{\mathrm{d}\boldsymbol{M}_j}{\mathrm{d}t} = \boldsymbol{0}$$

where $j = 1, 2$. $K^{\mathrm{s}}{}_j$ is the interfacial anisotropy parameter, $g_{\mathrm{r}j}$ is the mixing conductance on the j-th external interface, and $\hat{\boldsymbol{n}}_j$ is normal vector to the j-the external interface. J_{s} is the spin current absorbed at the interface.

The mixing conductance, spin current and interfacial anisotropy are assumed the be nonzero only on the **top interface**.

Internal interfaces

On the internal interfaces the Hoffman boundary conditions hold

$$\frac{2A_j}{M_{\mathrm{s}j}}\boldsymbol{M}_j \times \frac{\partial \boldsymbol{M}_j}{\partial \hat{\boldsymbol{n}}_j} - \frac{2K^{\mathrm{i}}{}_j}{M_{\mathrm{s}j}}(\boldsymbol{M}_j\cdot\hat{\boldsymbol{n}}_j)(\boldsymbol{M}_j \times \hat{\boldsymbol{n}}_j) - \frac{2A_{12}}{M_{\mathrm{s}i}}\boldsymbol{M}_j \times \boldsymbol{M}_i + \frac{\hbar\, g_{\mathrm{r}}}{4\pi}\left(\frac{\boldsymbol{M}_j}{M_{\mathrm{s}j}} \times \frac{\mathrm{d}\boldsymbol{M}_j}{\mathrm{d}t} - \frac{\boldsymbol{M}_i}{M_{\mathrm{s}i}} \times \frac{\mathrm{d}\boldsymbol{M}_i}{\mathrm{d}t}\right) = \boldsymbol{0}$$

where $j, i = 1, 2$, and $j \neq i$. $K^{\mathrm{i}}{}_j$ is the interfacial anisotropy parameter, and $\hat{\boldsymbol{n}}_j$ is normal vector to the j-th internal interface, while g_{r} is the mixing conductance of both internal interfaces $g_{\mathrm{r}}{}^{-1} = g_{\mathrm{r}}{}_{i1}^{-1} + g_{\mathrm{r}}{}_{i2}^{-1}$. The last term stands for the dynamic coupling of the magnetic layers.

Continuity of the dipolar-exchange field

Finally, the tangential component of $\boldsymbol{h}_j(\boldsymbol{r})$ and the normal component of $\boldsymbol{h}(\boldsymbol{r}) + \boldsymbol{m}(\boldsymbol{r})$ must be continuous across the interfaces. This leads to 4 more boundary conditions.

Method

The linearized LLG together with the Maxwell equations have solutions in the form

$$m_{j,x}(\boldsymbol{r}) = m_{j,x}(x)\,e^{i\boldsymbol{q}\cdot\boldsymbol{s}}, \quad m_{j,y}(\boldsymbol{r}) = m_{j,y}(x)\,e^{i\boldsymbol{q}\cdot\boldsymbol{s}}, \quad \psi_j(\boldsymbol{r}) = \psi_j(x)\,e^{i\boldsymbol{q}\cdot\boldsymbol{s}}$$

where $\boldsymbol{q} = (q_y, q_z)$ is the in-plane wave vector, and $\boldsymbol{s} = (y, z)$ is the in-plane coordinate, and

$$m_{j,x}(x) = m_{j,x}\,e^{ik_jx}, \quad m_{j,y}(x) = m_{j,y}\,e^{ik_jx}, \quad \psi_j(x) = \psi_j\,e^{ik_jx}$$

In case of Voigt geometry (when $q_z = 0$) they have nontrivial solutions for

$$k_{j,1}^2 = -q_y^2$$
$$k_{j,2(3)}^2 = -\frac{\mu_0 M_{\mathrm{s}j}^2}{2A_j}\left[(1/2 - i\,\alpha_j\,f_j) + \frac{H_0}{M_{\mathrm{s}j}} \pm \sqrt{f_j^2 + (1/2)^2}\right] - q_y^2$$

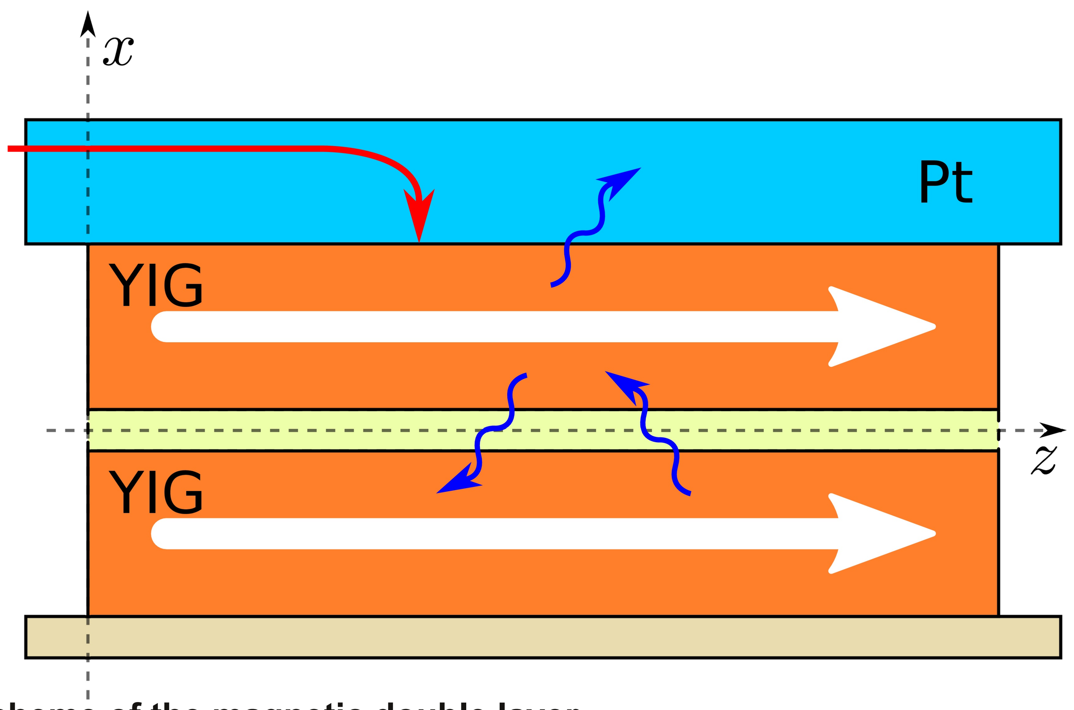
where $f_j = \omega/(\gamma_j\,\mu_0\,M_{\mathrm{s}j})$. Thus, the general solutions for $m_{j,x}$, $m_{j,y}$ and ψ_j can be written in the form

$$m_{j,x}(x) = \sum_{l=1}^{3}\left[C_{j,1}^{(l)}\cos(k_{j,l}x) + D_{j,1}^{(l)}\sin(k_{j,l}x)\right]$$
$$m_{j,y}(x) = \sum_{l=1}^{3}\left[C_{j,2}^{(l)}\cos(k_{j,l}x) + D_{j,2}^{(l)}\sin(k_{j,l}x)\right]$$
$$\psi_j(x) = \sum_{l=1}^{3}\left[C_{j,3}^{(l)}\cos(k_{j,l}x) + D_{j,3}^{(l)}\sin(k_{j,l}x)\right]$$

where j is layer index and l is index of the k-vector.

Inserting the latter solution into the boundary conditions we obtain 12×12 matrix, $\bar{\boldsymbol{M}}$, which determinant becomes zero at the resonance frequency $\omega = \omega_{\mathrm{r}}$

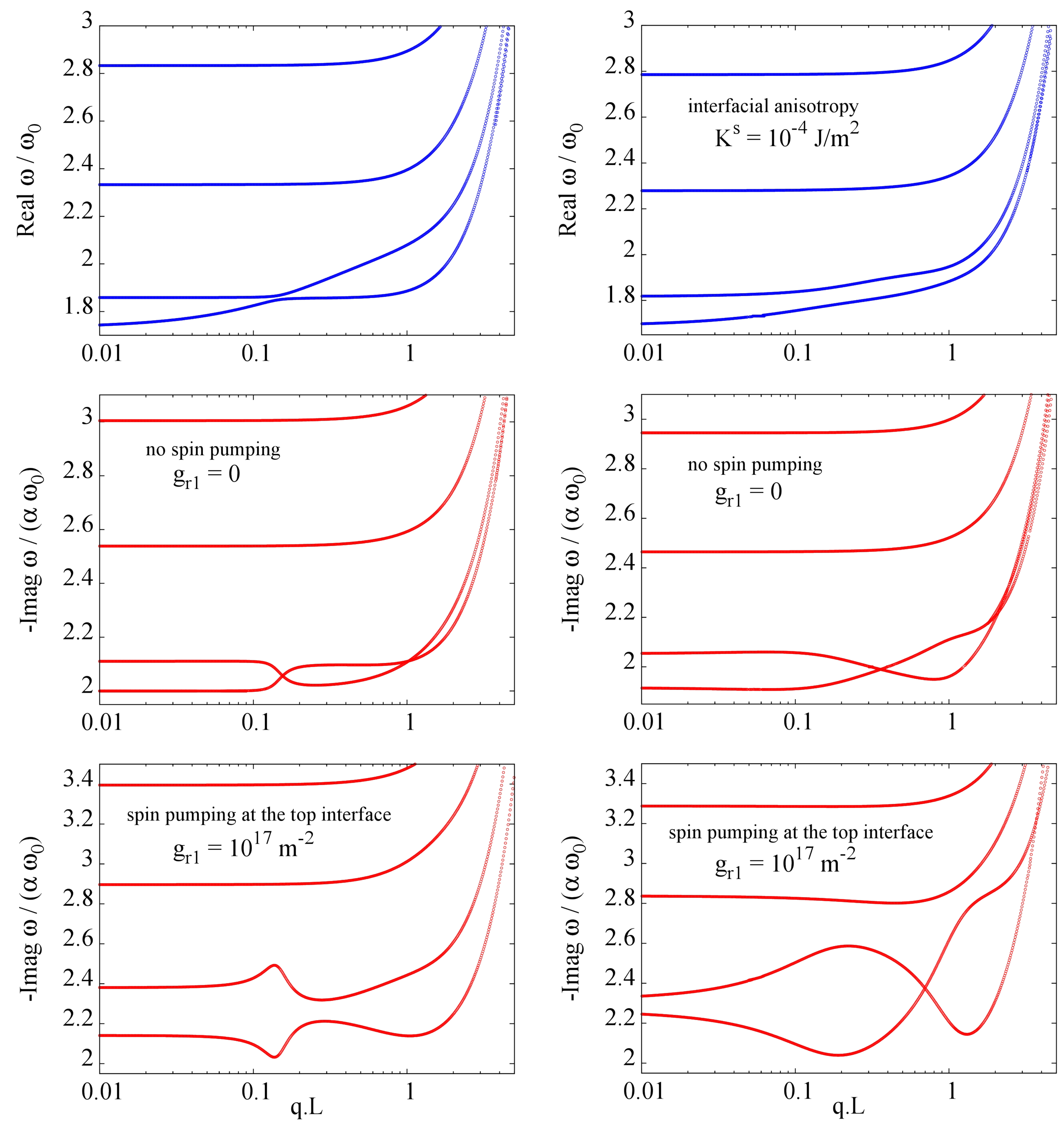
$$\det \bar{\boldsymbol{M}}(\omega_{\mathrm{r}}) = 0$$



Scheme of the magnetic double layer.
Two insulating magnetic layer of the thickness L=100 nm are separated by a thin nonmagnetic layer. Expect of magnetostatic field and interlayer exchange (RKKY) coupling, the magnetizations are dynamically coupled by due to the interlayer spin pumping (blue arrows) governed by the interlayer mixing conductance gr.
Due to the inverse Spin Hall Effect in the top Pt layer a current voltage along the layer results in the spin current Js (red line), which is absorbed at the top interface. Moreover, when the top magnetization changes, the spin current is emited through the top interface int Pt layer due to spin pumping (blue arrow).

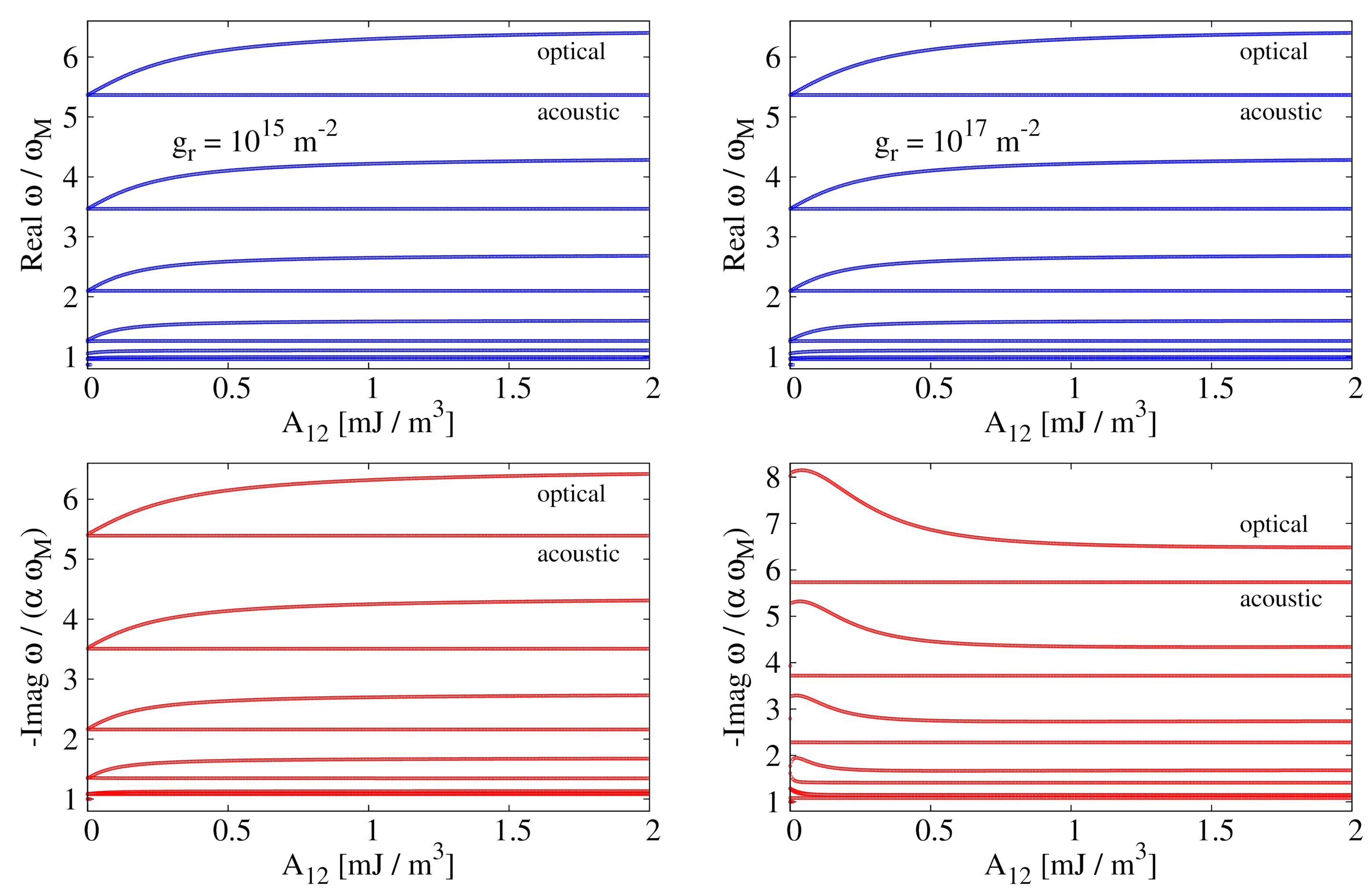
Parameters used in the calculations

Parameter	Abbreviation	Values
Saturated magnetization	$M_{\mathrm{s}j}$	1.56×10^5 A/m
Gilbert damping	α_j	6.7×10^{-5}
Exchange parameter	A_j	4.17×10^{-12} J/m^2
magnetic field	H_0	$M_{\mathrm{s}}/2$ (P config.) 0 (AP config.)
Layers' thicknesses	L_j	100 nm
Interlayer coupling	A_{12}	2×10^{-4} J/m^3 (P config.) -5×10^{-4} J/m^3 (AP config.)

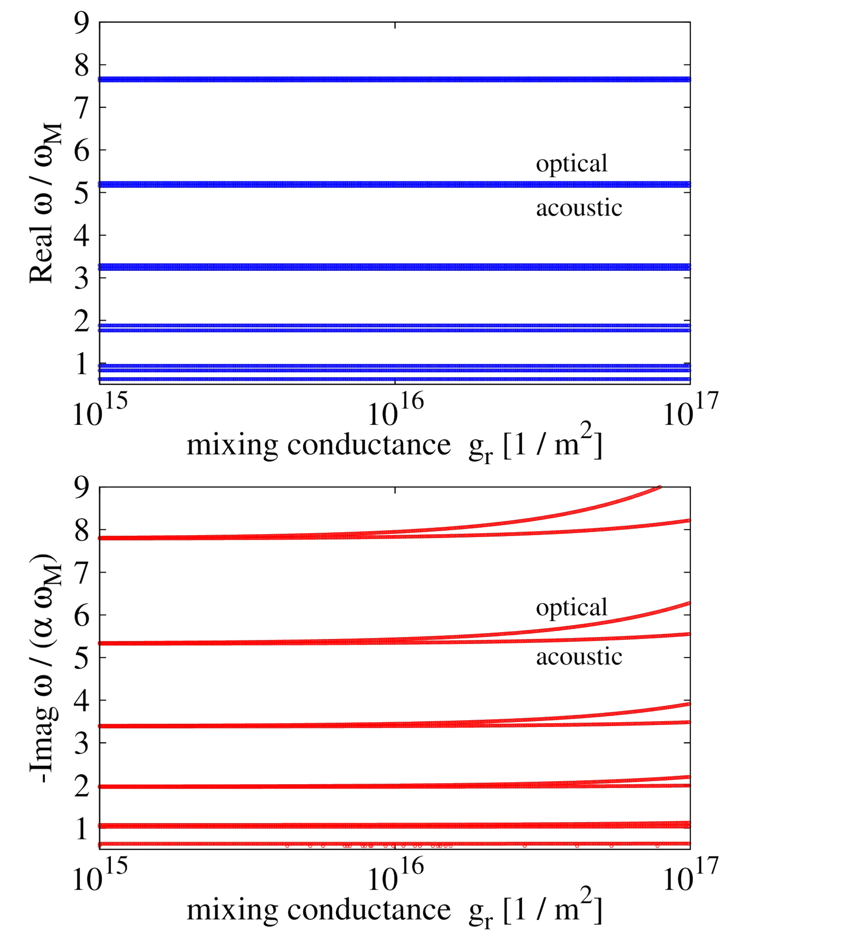


Four lower modes of the spin wave spectra in the parallel configuration as a function of in-plane wave vector q.
Left column: shows results without interfacial anisotropy.
Right column: show spin wave spectra with perpendicular anisotropy at the top interface.
Imaginary parts of the eigenferquencies are shown in case of zero an nonzero spin pumping at the top interface. The real parts do not change with g_{r1}.
In the calculations above, the interlayer mixing conductance was zero.

Real and imaginary parts of the spin wave eignefrequencies in **parallel configuration** as a function of interlayer exchange stiffness parameter for two dirrerent interlayer mixing conductances. Since the real parts (energies) are not influenced by the interlayer mixing conductance, the imaginary parts (spin wave life times) changes at higher g_r. Interestingly, the optical spin waves seems to be stronger influenced by the interlayer dynamic coupling.



Real and imaginary parts of the spin wave eignefrequencies in **parallel configuration** as a function of interlayer mixing conductance. Real parts do not depend on g_r. Imaginary parts of the acoustic spin waves change just slightly in comparison to the ones of the optical spin waves.



Similar effects as in the case of parallel configuration can be also observed in the **antiparallel** one.
In this case, the different variation of the spin wave life times with g_r is even more pronounced.