SWISS CONTRIBUTION

Polish-Swiss Research Programme

Theory of Spin Diode Effect

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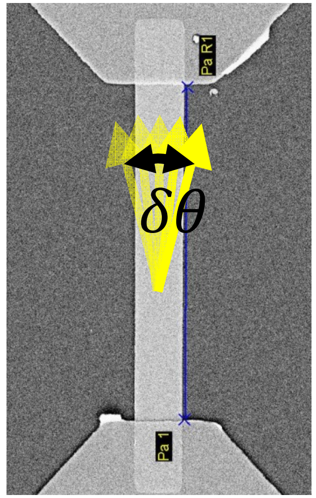
Outline:

- What is the spin diode effect ?
- Analytical model
- Applications: TMR/GMR/AMR structures
- Extensions of the model – GMR nanowire
- Summary

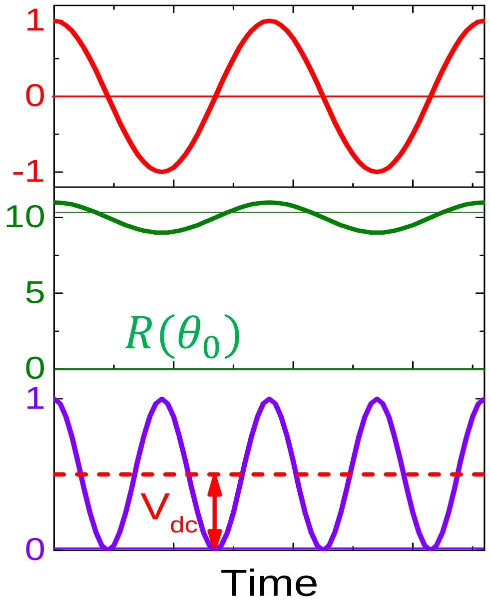
What is the spin diode effect ?

The idea is very simple (A. Tulapurkar et al., Nature 438, 330-342 (2005))

- magnetoresistive element: AMR, GMR, TMR



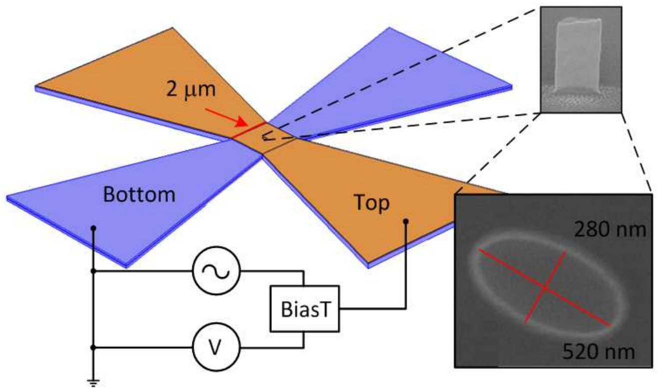
Then we pass the a.c. current through this element
If the current is the source of driving force for
magnetization dynamics then we can measure Vdc
(spin diode voltage) that results from the mixing
of a.c. current and oscillating resistance of the element.



RF current: $\boldsymbol{I_{a.c.}}$

Oscillating resistance: **R(t)**

Spin diode DC voltage: $\boldsymbol{V_{out} = I_{a.c.} \cdot R(t)}$



Model (1)

What is measured ? Output voltage:

$$V_{\text{out}} = V_{\text{dc}} + V_{\text{ac}} = \boxed{\frac{1}{R(\theta_0)}\mathcal{R}\{Ve^{i\omega t}\}}\boxed{\mathcal{R}\{\delta\bar{R}e^{i(\omega t+\beta)}\}}$$

a.c. current — **oscillating resistance**

$$V_{\text{dc}} = \frac{V}{2}\frac{\boxed{\delta\bar{R}}}{R(\theta_0)}\cos\beta.$$

The amplitude of resistance changes: $\overline{\delta R}$

$$R(\theta) = R_P + \frac{R_{AP} - R_P}{2}\left(1 - cos(\theta)\right)$$

TMR, GMR structures:
θ – the angle between two magnetic moments

$$R(\theta) = R_\perp + (R_\parallel - R_\perp)\cos^2\theta$$

$(\Delta R \equiv R_\parallel - R_\perp; R_{AP} - R_P)$

AMR structures:
θ – the angle between magnetic moment and current density vector

Model (2)

AMR:

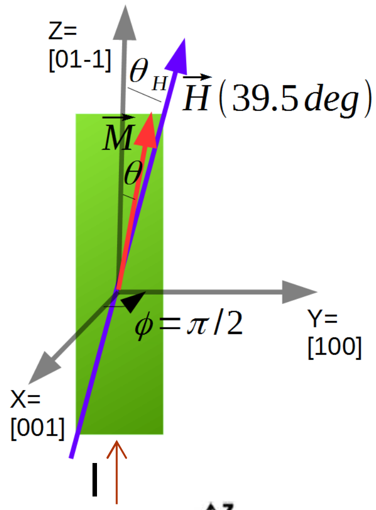
$$\delta R = -2\Delta R \sin\theta_0 \cos\theta_0 \delta\theta$$

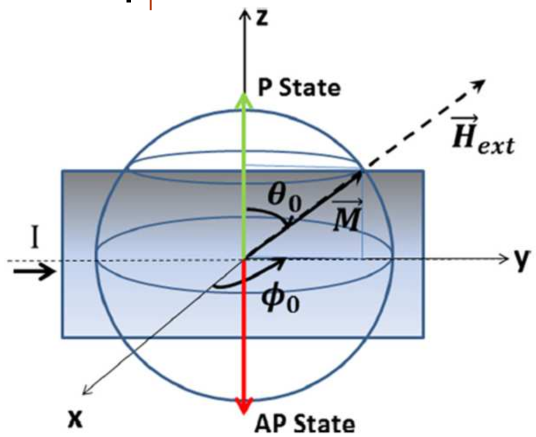
$$V_{dc} = -\frac{\eta V \Delta R}{R(\theta_0)} \sin\theta_0 \cos\theta_0 \,\Re\{\delta\theta\}$$

TMR /GMR:

$$\delta R = \frac{1}{2}\Delta R \sin\theta_0 \delta\theta$$

$$V_{dc} = -\frac{\eta V \Delta R}{4\, R(\theta_0)} \sin\theta_0 \,\Re\{\delta\theta\}$$





The goal is to calculate $\delta\theta$ for specific system

$\delta\theta$ may be derived from LLG/LLGS equation

Model (3) Calculations $\delta\theta$:

$$\frac{d\vec{M}}{dt} = -\gamma_e \vec{M} \times \vec{H}_{eff} + \frac{\alpha}{M_S}\vec{M} \times \frac{d\vec{M}}{dt} + \gamma_e \vec{\tau}$$

$$\vec{\boldsymbol{H}}_{\boldsymbol{eff}} = -\nabla\mathbf{U}$$

$U =$ — **Magnetic energy:**

U_a — **magneto--crystalline anisotropy**

$+ U_s$ — **shape anisotropy**

$+ U_z$ — **Zeeman-like Interactions**

$+ U_{coup}$ — **Interlayer coupling**

$+ U_{ms}$ — **Magnetoelastic energy**

$+ U_{Oe}$ — **Interaction with Oersted field (in the case: $\nabla \times \vec{H}_{oe} = 0$)**

STT term in standard form for multilayer systems i.e. $\tau_{\parallel}\, \vec{M}_1 \times (\vec{M}_1 \times \vec{M}_2) + \tau_{\perp}(\vec{M}_1 \times \vec{M}_2)$ or for non-uniform magnetization distribution:

$$-(\vec{u}(\vec{r}) \cdot \nabla)\vec{m}(\vec{r}) + \beta\, \vec{m}(\vec{r}) \times [(\vec{u} \cdot \vec{\nabla})\vec{m}(\vec{r})]$$

(magnetization distribution within the sample has to be assumed), Such a torque effectively acts on averaged magnetic moment of the sample $\vec{M}$ (macrospin with nonhomogenities..)

Model (4)

LLG equation in spherical coordinates with STT Slonczewski-like and field-like terms:

$$(\sin\theta\dot{\phi}\hat{e}_\phi + \dot{\theta}\hat{e}_\theta) - \alpha\sin\theta\dot{\phi}\hat{e}_\theta + \frac{\alpha}{S}\dot{\theta}\hat{e}_\phi =$$

$$= -\gamma_e\frac{\partial U}{\partial\theta}\hat{e}_\phi + \frac{\gamma_e}{\sin\theta}\frac{\partial U}{\partial\phi}\hat{e}_\theta + \gamma_e\frac{\tau_\parallel}{S}\sin\theta\hat{e}_\theta - \gamma_e\frac{\tau_\perp}{S}\sin\theta\hat{e}_\phi$$

We assume that magnetic moment oscillates harmonically around stationary point, so the solutions for two angles describing $\vec{M}$ are in the form:

$$\theta(\mathrm{t}) = \theta_0 + \delta\theta e^{\mathrm{i}\omega \mathrm{t}} \qquad \phi(\mathrm{t}) = \phi_0 + \delta\phi e^{\mathrm{i}\omega \mathrm{t}}$$

$$\begin{pmatrix} \mathrm{i}\omega\delta\theta(\mathrm{t}) \\ \mathrm{i}\omega\delta\phi(\mathrm{t}) \end{pmatrix} = \begin{pmatrix} \frac{\gamma_e}{1+\alpha^2}\left(\frac{1}{\sin\theta}\frac{\partial U}{\partial\phi} - \alpha\frac{\partial U}{\partial\theta} + \frac{\sin\theta}{S}(-\tau_\parallel - \alpha\tau_\perp)\right) \\ \frac{\gamma_e}{1+\alpha^2}\left(-\frac{1}{\sin\theta}\frac{\partial U}{\partial\theta} - \frac{\alpha}{\sin^2\theta}\frac{\partial U}{\partial\phi} - \frac{1}{S}(-\alpha\tau_\parallel + \tau_\perp)\right) \end{pmatrix}$$

Model (5)

Linearization of RHS

$$
\begin{pmatrix} \delta\theta \\ \delta\phi \end{pmatrix} = \frac{-\gamma_e}{\Gamma(\omega^2 - \omega_0^2 - i\omega\sigma)} \times
$$

$$
\times \left(\begin{array}{c}
\frac{\gamma_e}{S}(B\alpha - C\csc\theta)\left(S\csc\theta(M + \alpha H\csc\theta) + \frac{\tau_\perp}{dV} - \alpha\frac{\tau_\parallel}{dV}\right) + \\
+ \left[i(1+\alpha^2)\omega + \gamma_e\csc\theta(B + C\alpha\csc\theta)\right] \\
\left[-\alpha M + H\csc\theta + \frac{\sin\theta}{S}\left(-\frac{d\tau_\parallel}{dV} - \alpha\frac{d\tau_\perp}{dV}\right)\right] \\
\\
\frac{1}{S}\left(\left[-\frac{d\tau_\perp}{dV} + \alpha\frac{d\tau_\parallel}{dV} - S\csc\theta\,(M + H\alpha\csc\theta)\right] \times \right. \\
\times \left[i(1+\alpha^2)\omega + \gamma_e\left(D\alpha - \frac{\cos\theta}{S}\left(-\tau_\parallel - \alpha\tau_\perp\right) + \frac{1}{\sin\theta}\left(\frac{\cos\theta}{\sin\theta}A - B\right)\right)\right] - \\
-\frac{\gamma_e}{\sin\theta}\left[D + B\alpha\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\left(E + 2\alpha A\frac{1}{\sin\theta}\right)\right] \\
\left. \cdot \left[HS\frac{1}{\sin\theta} - MS\alpha + \sin\theta\left(-\frac{d\tau_\parallel}{dV} - \alpha\frac{d\tau_\perp}{dV}\right)\right] \right)
\end{array} \right) V_p e^{i\Psi}
$$

Ψ - phase shift between $\vec{M}$ and driving-force (a.c. current)

$$A \equiv \frac{1}{S}\frac{\partial U}{\partial \phi}, B \equiv \frac{1}{S}\frac{\partial^2 U}{\partial \phi \partial \theta}, C \equiv \frac{1}{S}\frac{\partial^2 U}{\partial \phi^2}, D \equiv \frac{1}{S}\frac{\partial^2 U}{\partial \theta^2}, E \equiv \frac{1}{S}\frac{\partial U}{\partial \theta}, H \equiv \frac{1}{S}\frac{\partial^2 U}{\partial \phi \partial V}, M \equiv \frac{1}{S}\frac{\partial^2 U}{\partial \theta \partial V}$$

$$\Gamma = (1+\alpha^2)^2$$

$$\omega_0^2 = \frac{1}{\mathrm{S}}\frac{\gamma_e^2}{\Gamma \sin\theta}\left[-\frac{\mathrm{S}}{\sin\theta}(\mathrm{B}^2 - \mathrm{CD})(1+\alpha^2) + \cos\theta\left(\mathrm{B}[\alpha\tau_\perp + \tau_\parallel] + \frac{\alpha}{\sin\theta}[\mathrm{EBS} - \mathrm{C}(-\tau_\parallel - \tau_\perp\alpha)]\right] + \frac{\mathrm{S}}{\sin^2\theta}[\mathrm{AB}(1+2\alpha^2) - \mathrm{CE}] - \frac{\mathrm{S}\alpha}{\sin^3\theta}\mathrm{AC}\right)\right] \tag{D.3}$$

$$\sigma = \frac{1}{1+\alpha^2}\frac{\gamma_e}{4\mathrm{S}\sin^2\theta}\left(2\mathrm{S}\alpha[2\mathrm{C} + \mathrm{D} - \mathrm{D}\cos 2\theta] + \cos\theta[4\mathrm{SA} + \tau_\parallel + \alpha\tau_\perp] + \cos 3\theta[-\tau_\parallel - \alpha\tau_\perp]\right) \tag{D.4}$$

if dynamics is driven by Oersted field, we use derivatives with respect to $\vec{H}_{oe}$ (instead of V_P)

$$\mathcal{R}(\delta\theta) = \frac{-\gamma_e V_p}{\Gamma((\omega^2 - \omega_0^2)^2 + \omega^2\sigma^2)}(\Omega\cos\Psi - \Sigma\sin\Psi)$$

$$\mathcal{R}(\delta\theta) = \frac{-\gamma_e V_p}{\Gamma((\omega^2-\omega_0^2)^2+\omega^2\sigma^2)}\left(\Omega\cos\Psi - \Sigma\sin\Psi\right)$$

$$\Omega \equiv (1+\alpha^2)\left[(\omega^2-\omega_0^2)\gamma_e\csc^2\theta\left[\mathrm{HB}-\mathrm{CM}-\frac{\mathrm{B}}{\mathrm{S}}\sin^2\theta\frac{\mathrm{d}\tau_\parallel}{\mathrm{dV}}-\frac{\mathrm{C}}{\mathrm{S}}\sin\theta\frac{\mathrm{d}\tau_\perp}{\mathrm{dV}}\right]+\right.$$

$$\left.\sigma\omega^2\left[\frac{H}{\sin\theta}-\alpha M+\frac{\sin\theta}{S}\left(-\frac{d\tau_\parallel}{dV}-\alpha\frac{d\tau_\perp}{dV}\right)\right]\right] \qquad (11.10)$$

$$\Sigma \equiv -(1+\alpha^2)\left[\sigma\omega\gamma_e\csc^2\theta\left[\mathrm{HB}-\mathrm{CM}-\frac{\mathrm{B}}{\mathrm{S}}\sin^2\theta\frac{\mathrm{d}\tau_\parallel}{\mathrm{dV}}-\frac{\mathrm{C}}{\mathrm{S}}\sin\theta\frac{\mathrm{d}\tau_\perp}{\mathrm{dV}}\right]+\right.$$

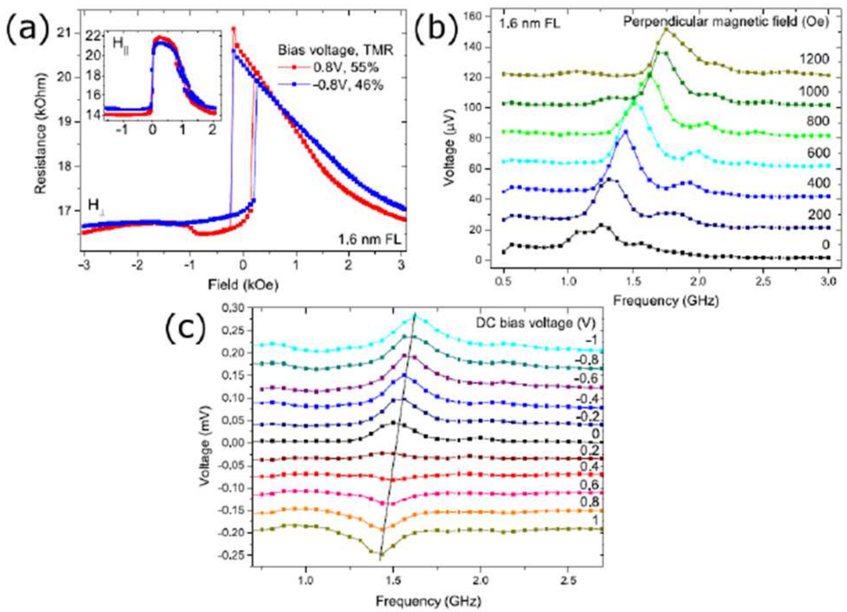
$$\left.-\omega(\omega^2-\omega_0^2)\left[\frac{H}{\sin\theta}-\alpha M+\frac{\sin\theta}{S}\left(-\frac{d\tau_\parallel}{dV}-\alpha\frac{d\tau_\perp}{dV}\right)\right]\right] \qquad (11.11)$$

[green box] - symmetric contribution to the Vdc lineshape

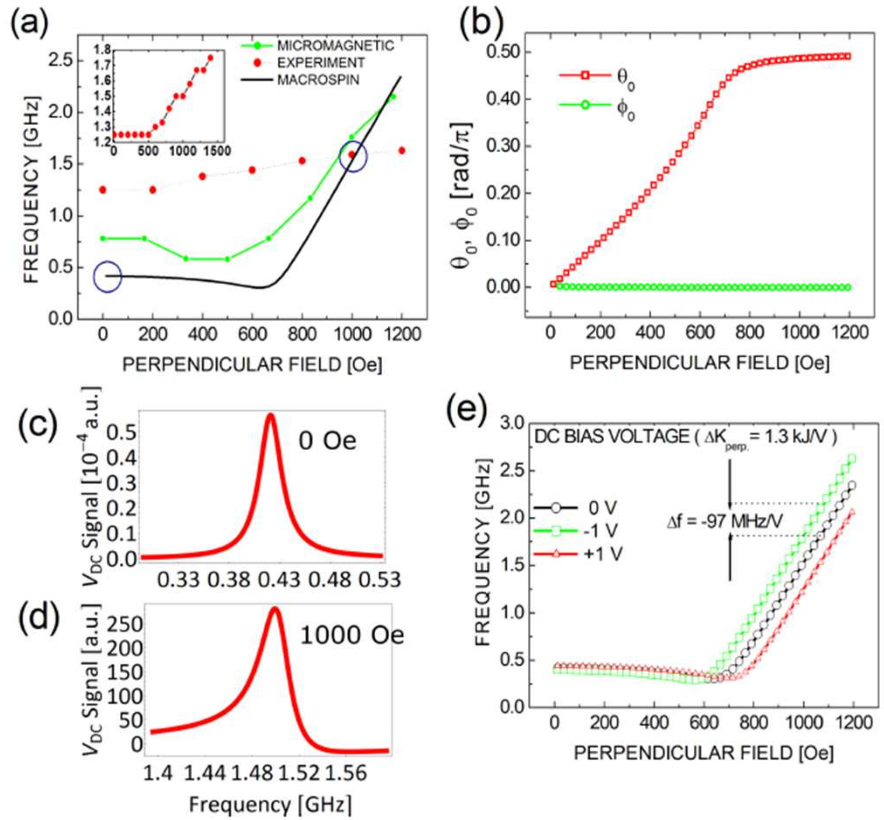
[red box] - antisymmetric contribution to the Vdc lineshape

Applications(1):

TMR nanopillar with perpendicular anisotropy – first comparison with the experiment.



Rysunek 11.1. (a) Rezystancja w funkcji przyłożonego pola magnetycznego w kierunku prostopadłym do płaszczyzny próbki dla dwóch polaryzacji napięcia stałego: +0.8 V i -0.8 V. Inset: wynik dla pola przyłożonego w płaszczyźnie próbki. (b) sygnał FMR mierzony za pomocą spinowego efektu diodowego dla różnych wartości prostopadłego do próbki pola magnetycznego (przy braku napięcia stałego). (c) sygnał FMR zmierzony w funkcji napięcia stałego przy ustalonej wartości prostopadle skierowanego zewnętrznego pola magnetycznego (600 Oe). (źródło: [168])



Energy with perpendicular anisotropy:

$$U = K_{\perp 1}[\cos^2\theta + \sin\theta\,\sin\phi\,] - M_s\left(\vec{H}_{ext}\cdot\hat{e}_M - \frac{M_s}{2\mu_0}\hat{e}_M^T\hat{N}\hat{e}_M\right)$$

$K_{\perp 1}$-is V(bias voltage)dependent

Application (2) – GMR structure in CIP configuration

Ziętek et al., Phys. Rev. B 91, 014430 (2015)

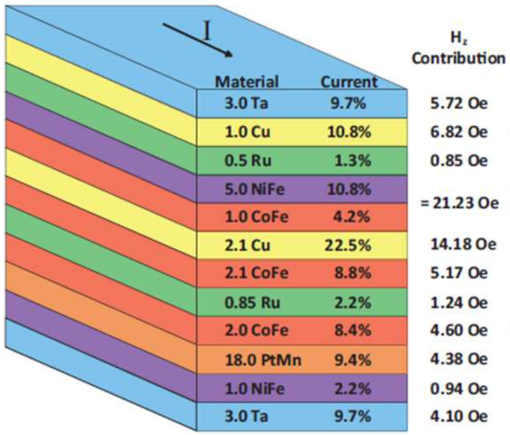


FIG. 2. (Color online) Current distribution in the GMR stack and magnetic field contribution from each layer. Note that the Oersted field contribution from the Cu spacer layer is dominant.

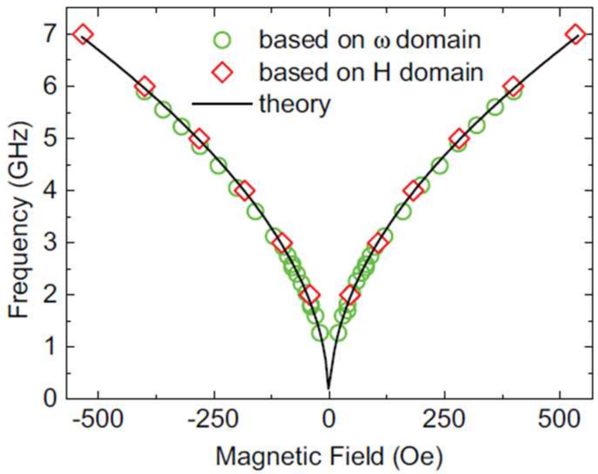


FIG. 5. (Color online) Dispersion relation of the FL magnetization for the SVGMR strip. The H domain denotes a measurement conducted under a sweeping magnetic field at a constant frequency, while the ω domain denotes a measurement conducted in a constant magnetic field with the frequency being swept.

$$U = K_{\parallel} \sin^2\theta - M_S \times \left(\vec{H}_{\text{ext}} \cdot \hat{e}_M - \frac{M_S}{2\mu_0} \hat{e}_M^T \hat{N} \hat{e}_M + H_{\text{Oe}} \hat{e}_z \cdot \hat{e}_M \right)$$

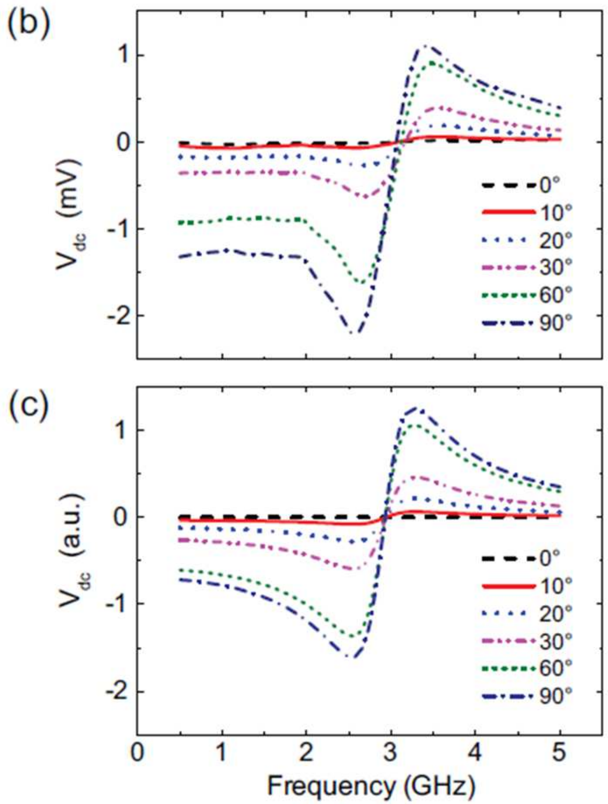
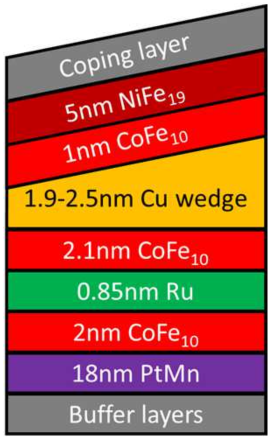


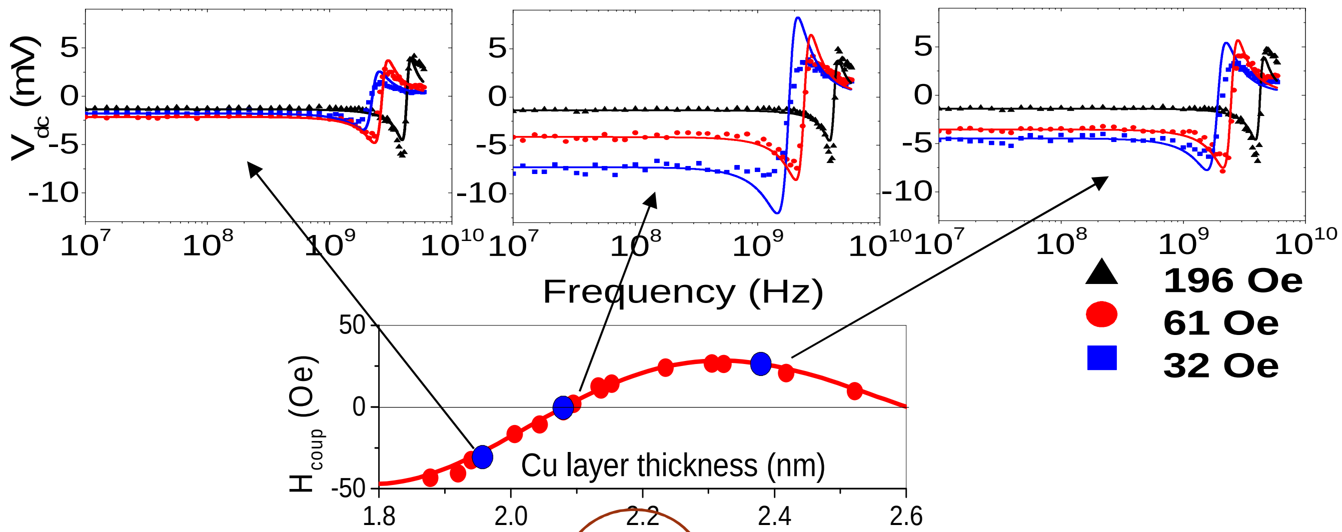
FIG. 7. (Color online) (a) The dc voltage originating from the spin diode effect as a function of the magnetic field angle θ and (b) frequency. (c) Theoretical spectra predicted by Eq. (21).

Application (3) – GMR sample in CIP configuration

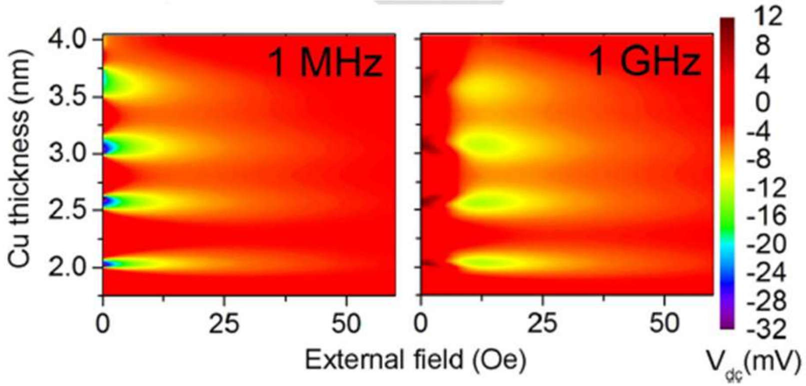
(the influence of IEC on the out-of-resonance Vdc signal driven by Oersted field)

Ziętek et al., APL 107, 122410 (2015)

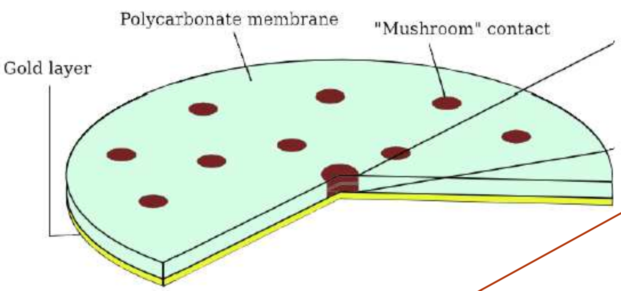


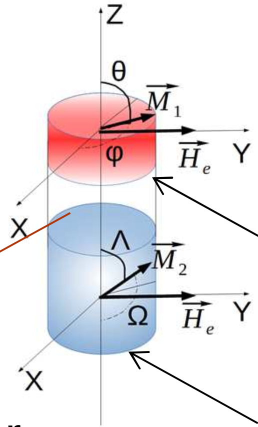


Additional energy term (IEC): $U_{coup} = -\vec{M} \cdot \vec{H}_{coup}$ → RKKY interaction



Extension of the spin diode model - GMR nanowires. Motivation :

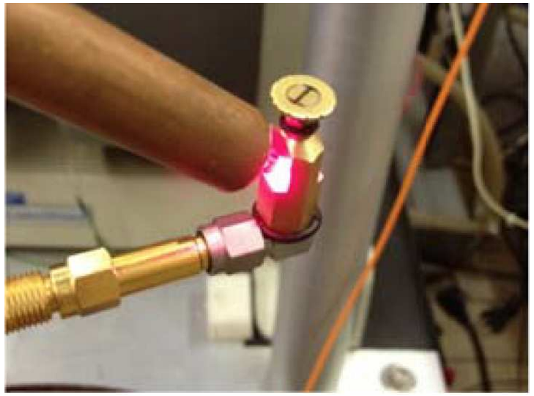


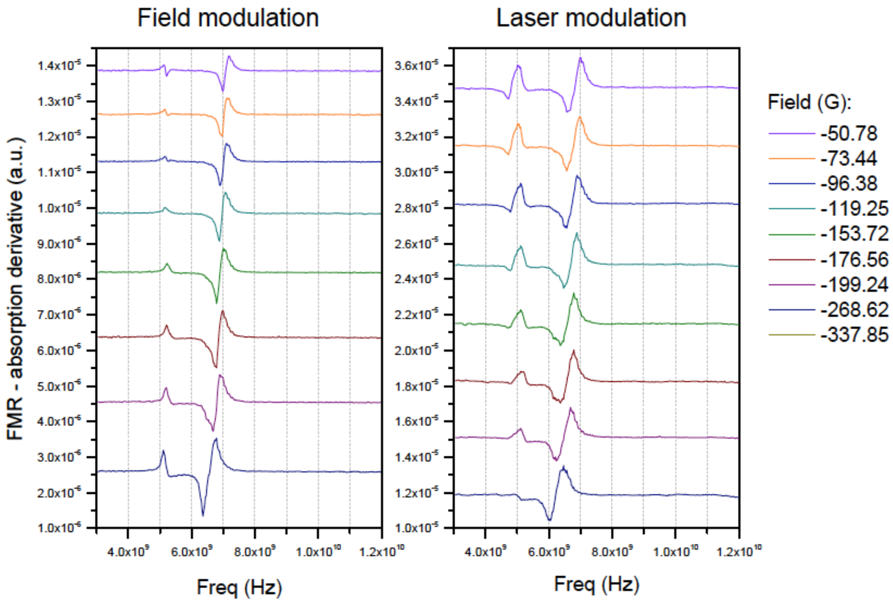


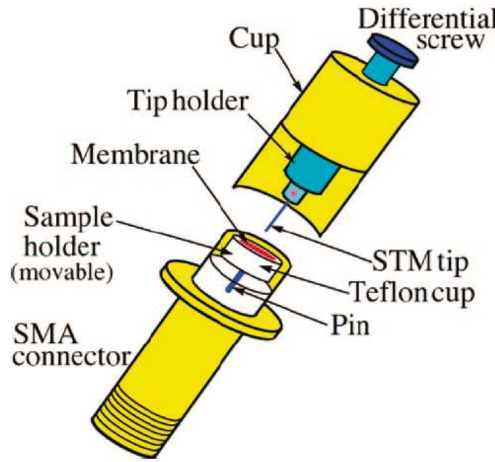
Cu (7 nm) spacer

Thin (5nm) magnetic (Co) layer

Thick(25nm) magnetic (Co) layer



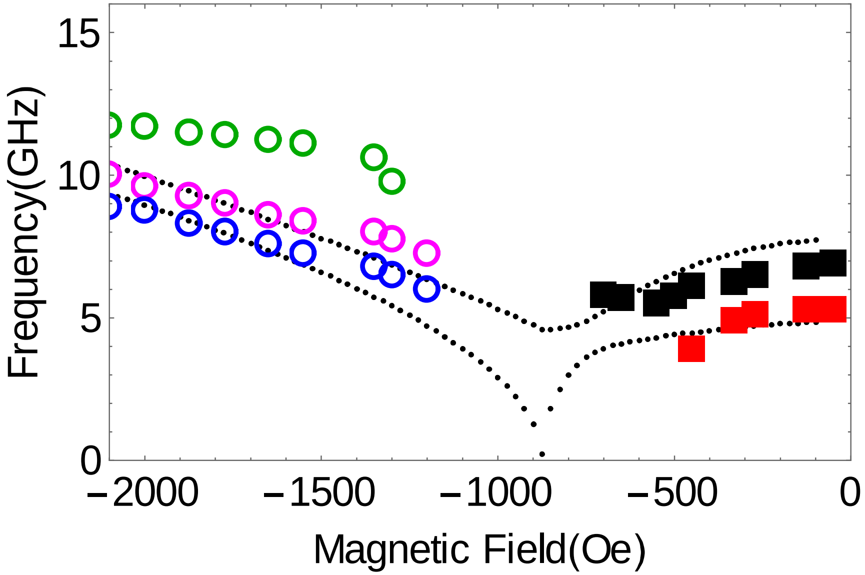


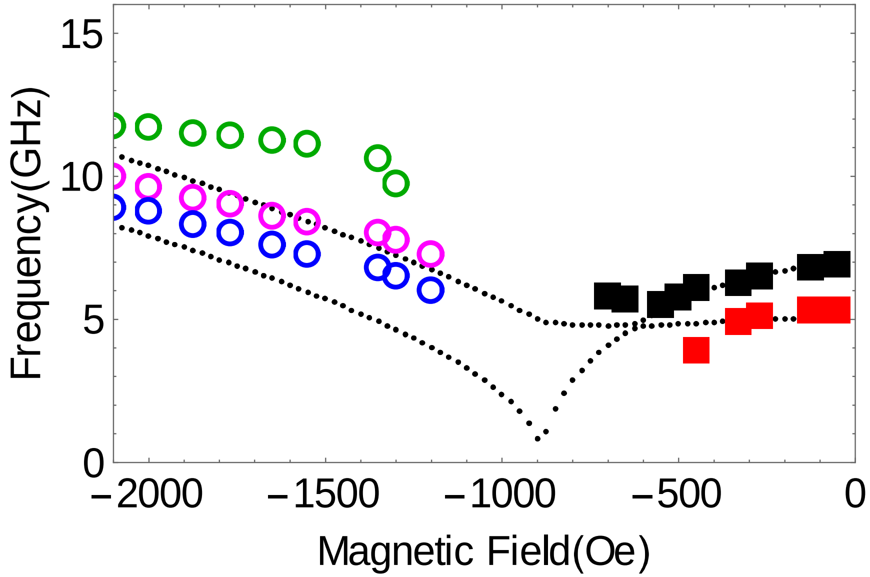


Results – relations of dispersion

Thin layer (fixed 5 nm)

The best fits:

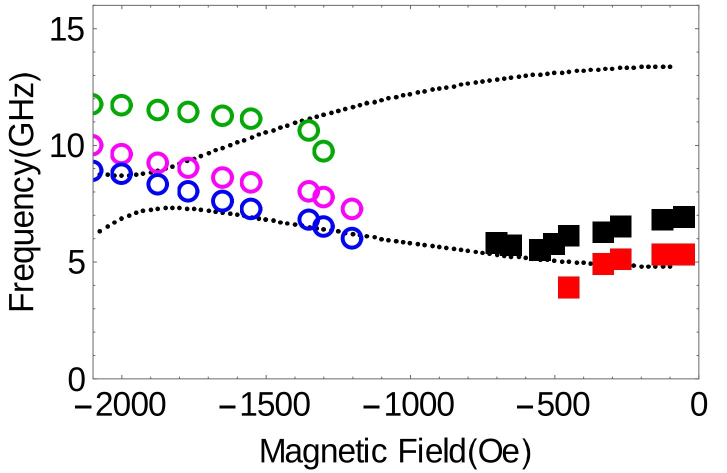




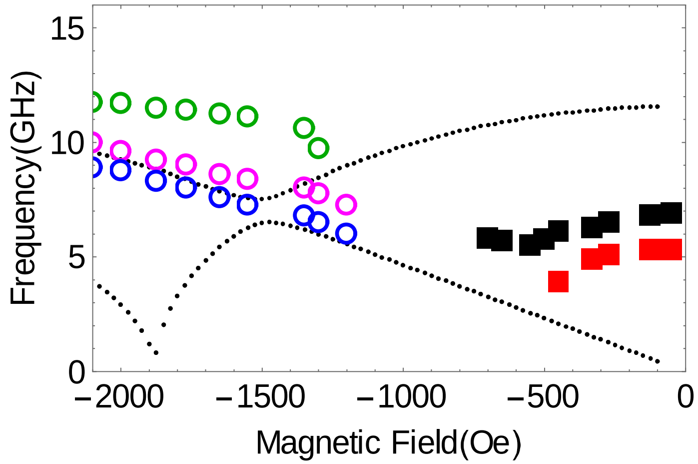
thick layer: d = 10 nm, K = 20 kJ/m3 , Ms= 1.4 T
thin layer: d = 5 nm, K = 35kJ/m3, Ms = 1.3 T
Spacer thickness**: 30 nm** (corresponding to dipolar coupling fields: 60 Oe (in thick layer) and 113 Oe (in thin layer)

thick layer: d = 10 nm, K = 35 kJ/m3 , Ms= 1.4 T
thin layer: d = 5 nm, K = 20kJ/m3, Ms = 1.3 T
Spacer thickness: 30 nm (corresponding to dipolar coupling fields: 60 Oe (in thick layer) and 113 Oe (in thin layer)

Thick layer = 10 nm

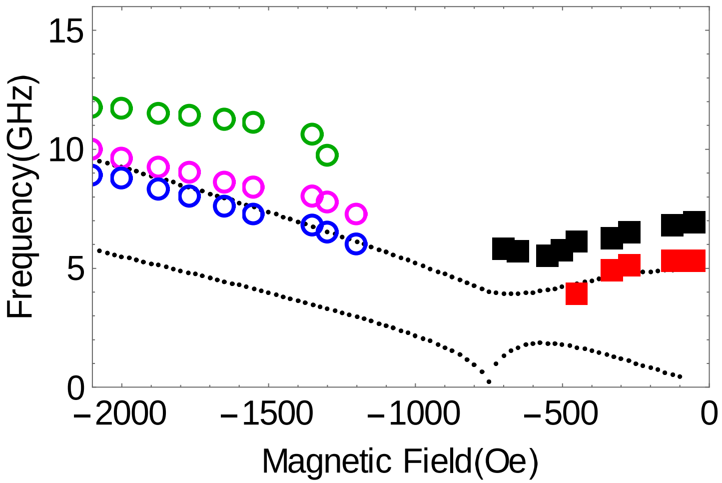


More realistic Cu thickness = 7 nm
dipolar fields are large...



and with reduced magnetocrystalline
anisotropy (in both layers ~0.1 kJ/m3)

More realistic: thick layer = 20 nm



Thick layer, and thin layer: K = 100 J/m3 Ms= 0.9 T

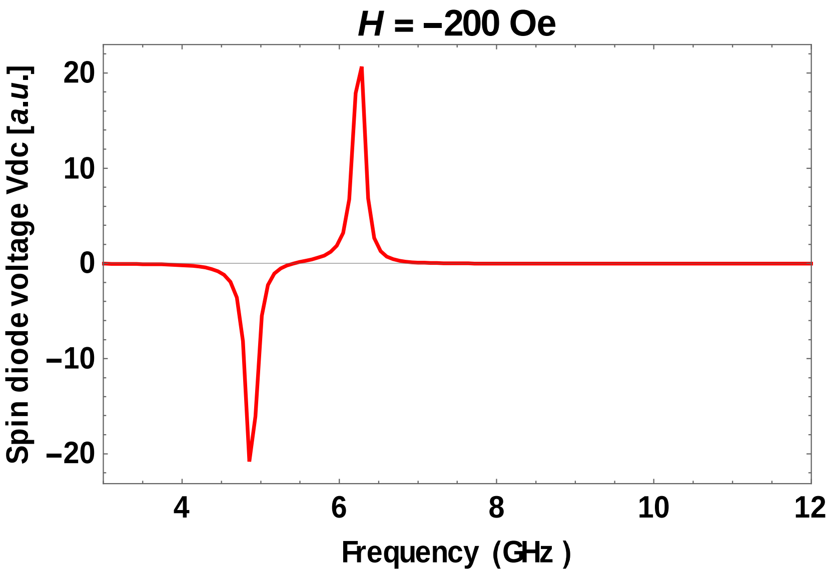
Best fit for:

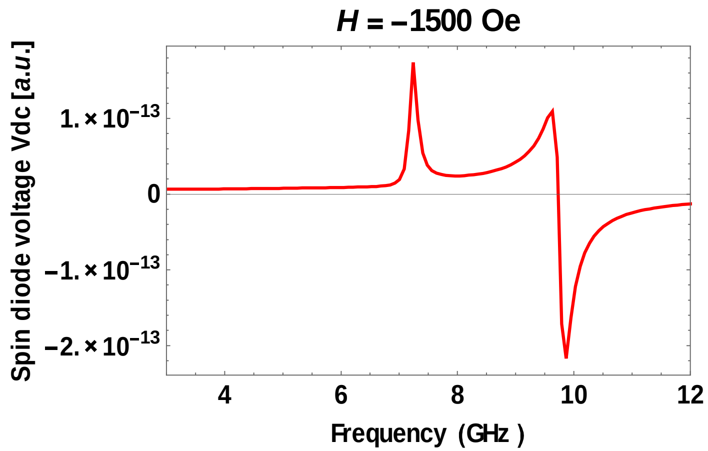
Spacer thickness: 17.5 nm (corresponding to dipolar coupling fields: 67 Oe (in thick layer) and 306 Oe (in thin layer)

<u>**The conclusion from f(H) within macrospin model:**</u>
The best fit is for thick layer (~10nm) and very thick Cu spacer (30 nm) with very high magnetocrystalline anisotropy (20-40kJ/m3) and high saturation magnetization (1.3 -1.4 T).

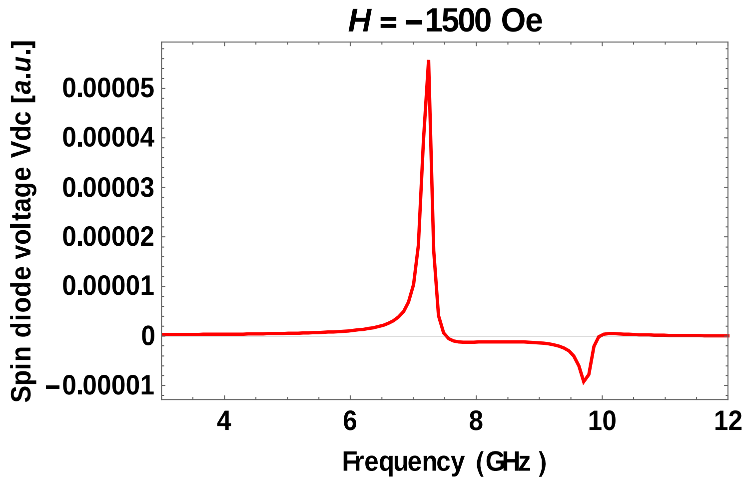
Vdc lineshapes
(for the best f(H) fit):

Dynamics driven only by STT:





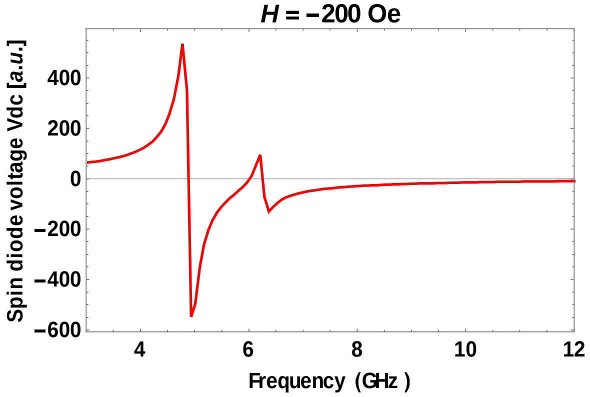
without coupling -> peaks still symmetric

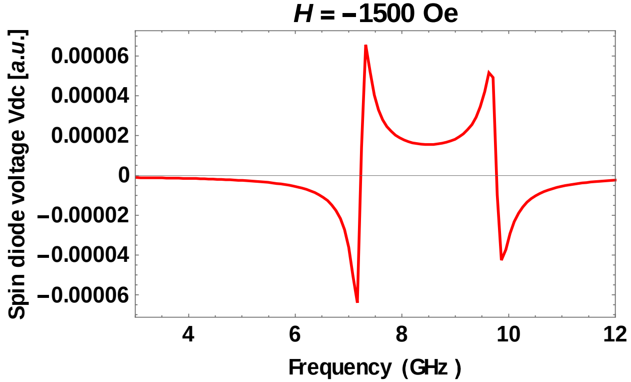


Vdc lineshapes

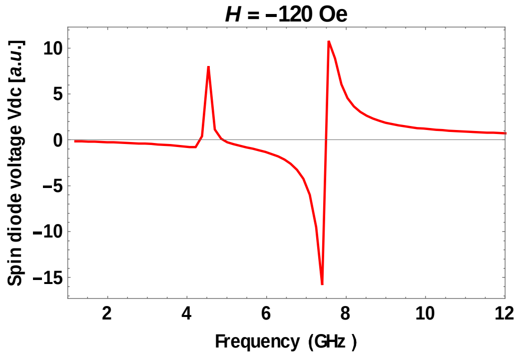
(for the best f(H) fit):

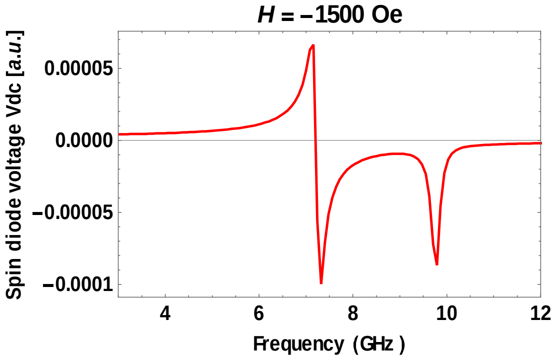
Dynamics driven only by Oersted field:





Playing with the parameters (phase shift, amplitudes of STT and Oersted field) makes possible to get lineshapes similar to experimental ones.





Summary (short):

- In some cases the model (macrospin) agrees well with the experimental results

- However, in the case of strongly coupled layers the theoretical analysis becomes very difficult because of number of free parameters