THEORY OF SPIN HALL EFFECT AND INTERFACE SPIN-ORBIT COUPLING

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NANOSPIN: Nanoscale spin torque devices for spin electronics
Summarizing MEETING, 11th-12th of July 2016

SWISS CONTRIBUTION
Polish-Swiss Research Programme
OUTLINE

- INTRODUCTION: SPIN ORBIT DRIVEN PHENOMENA
  - MICROSCOPIC MECHANISMS LEADING TO AHE/SHE
  - SPIN TORQUES DUE TO SPIN-ORBIT COUPLING
  - SHE AND CISP IN SEMICONDUCTOR HETEROSTRUCTURES WITH RASHBA SOI

- DETERMINATION OF SPIN-ORBIT TORQUES:
  THEORETICAL DESCRIPTION OF HARMONIC HALL VOLTAGE MEASUREMENTS

- PHENOMENOLOGICAL DESCRIPTION OF SOTs
  - DRIFT-DIFFUSION MODEL FOR Ta|CoFeB
    - literature – model for Ta|CoFeB by Kim et.al.
    - results for trilayer spin-diffusion model of Ta|interface|CoFeB
  - TRILAYER SPIN-DIFFUSION MODEL: SUMMARY
  - AHE IN Ta|CoFeB
  - ANOTHER APPROACHES
SPIN-ORBIT DRIVEN PHENOMENA

ANOMALOUS HALL EFFECT

CURRENT-INDUCED SPIN POLARIZATION

THERMAL CURRENT-INDUCED SPIN POLARIZATION

SPIN HALL EFFECT

SPIN-HALL EFFECT

SPIN NERNST EFFECT

SPIN HALL TORQUE

INVERSE SPIN HALL EFFECT

SPIN-ORBIT TORQUE

MICROSCOPIC MECHANISMS

- INTRINSIC CONTRIBUTION

BAND STRUCTURE TOPOLOGY

An external electric field leads to an interband coherence that gives rise to contribution to the transverse velocity – an anomalous velocity related to Berry phase curvature

\[ \nu_y = \frac{\partial \varepsilon_{n\mathbf{k}}}{\partial k_y} - eE_x \mathcal{F}_z \]

**anomalous velocity**

\[ \hat{H}_{SO} = -\frac{1}{4m^2c^2}\hat{s} \cdot (\hat{p} \times \nabla V) \]

\[ \hat{H}_E = eE \hat{\mathbf{\hat{r}}} \]

\[ \hat{H}_0 \longrightarrow \langle u_{n\mathbf{k}} | \hat{H}_E | u_{n\mathbf{k}} \rangle = ieE \langle u_{n\mathbf{k}} | \frac{\partial u_{n\mathbf{k}}}{\partial \mathbf{k}} \rangle \neq 0 \quad n \neq \bar{n} \]

\[ \hat{H} = \hat{H}_0 + eE_x \hat{\mathbf{\hat{r}}} \longrightarrow |u'_{n\mathbf{k}}\rangle = |u_{n\mathbf{k}}\rangle + ieE_x \sum_{\bar{n} \neq n} \langle u_{\bar{n\mathbf{k}}} | \frac{\partial u_{n\mathbf{k}}}{\partial \mathbf{k}_x} \rangle |u_{n\mathbf{k}}\rangle \]

\[ \Psi_{n\mathbf{k}e}(r, t) = e^{-i\hat{H}t} \Psi_{n\mathbf{k}e}(r) \]

\[ \Psi_{n\mathbf{k}e}(r) = \int \frac{d^Dk}{L^{D/2}} a(k)e^{i \{ \mathbf{k} \cdot \mathbf{r} \}} |u'_{n\mathbf{k}}\rangle \]

\[ \nu_y = \frac{d}{dt} \langle \Psi_{n\mathbf{k}e}^\dagger (r, t) | \hat{\mathbf{\hat{y}}} | \Psi_{n\mathbf{k}e}(r, t) \rangle = \langle u'_{n\mathbf{k}}\rangle - i \left[ i \frac{\partial}{\partial k_y}, H_0 \right] |u_{n\mathbf{k}}\rangle \]

\[ = \frac{\partial \varepsilon_{n\mathbf{k}}}{\partial k_y} + ieE_x \left( \langle \frac{\partial u_{n\mathbf{k}}}{\partial k_y} | \frac{\partial u_{n\mathbf{k}}}{\partial k_x} \rangle - \langle \frac{\partial u_{n\mathbf{k}}}{\partial k_x} | \frac{\partial u_{n\mathbf{k}}}{\partial k_y} \rangle \right) \]

Intrinsic spin Hall conductivity for 4d and 5d transition metals (microscopic tight-binding calculations);

\( \gamma \) – a quasiparticle damping rate

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**MICROSCOPIC MECHANISMS**

- **EXTRINSIC CONTRIBUTION**

- **SKEW SCATTERING**

- **SIDE JUMP**

- **EXTRINSIC SIDE-JUMP:** The contribution arising from the non-spin-orbit coupled part of the wave-packet scattering off the spin-orbit coupled disordered.

- **INTRINSIC SIDE-JUMP:** The contribution arising from the spin-orbit coupled part of the wave-packet formed by the Bloch electrons scattering off the scalar potential alone without spin-orbit coupling.

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**ANOMALOUS HALL EFFECT**

\[ \sigma_{xy} = \sigma_{xy}^{\text{int}} + \sigma_{xy}^{\text{sj}} + \sigma_{xy}^{\text{ss}} \]

\( \sim \tau^0 \) \quad \left( \sim \sigma_{xx}^0 \right)

\( \sim \tau^1 \) \quad \left( \sim \sigma_{xx}^1 \right)

**HIGH CONDUCTIVITY REGIME**

- \( \sigma_{xx} > 0.5 \times 10^6 \) (\( \Omega \text{ cm}^{-1} \))
- \( \sigma_{xy}^{\text{AH}} \sim \sigma_{xx} \)

**GOOD METAL REGIME**

- \( \sigma_{xx} \sim 10^4 - 10^6 \) (\( \Omega \text{ cm}^{-1} \))
- \( \sigma_{xy}^{\text{AH}} \sim \sigma_{xx}^0 \)

**BAD-METAL-HOPPING REGIME**

- \( \sigma_{xx} < 10^4 \) (\( \Omega \text{ cm}^{-1} \))
- \( \sigma_{xy}^{\text{AH}} \sim \sigma_{xx}^{1.6-1.8} \)

skew-scattering mechanism dominate

scattering-independent mechanisms or side jump dominate

there is currently no theory which predicts the observed scaling

N. Nagaosa et al., Rev. Mod. Phys. 82, 1539 (2010); J. Sinova et al., Rev. Mod Phys. 87, 1213 (2015)
CISP IN 2D ELECTRON GAS WITH RASHBA SOI

\[ H = \sum_k \psi_k^\dagger H_k \psi_k \]

\[ H_k = \frac{\hbar^2 k^2}{2m} + H_{RSO} \]

\[ H_{RSO} = \alpha(k_y \sigma_x - k_x \sigma_y) \]

\[ H_{SO} = \frac{\hbar}{2} \vec{B}_{eff}(\vec{k}) \cdot \vec{\sigma} \]

\[ \vec{B}_{eff}(\vec{k}) = \frac{2\alpha}{\hbar} (k_y, -k_x, 0) \]

\[ \vec{B}_{eff}(\vec{k}) \sim -\vec{k} \times \vec{E} \]

\[ \vec{E} = -\frac{1}{e} \nabla V(\vec{r}) \]

\[ \mu > 0 : \]

\[ \sigma_{xy}^{sz} = \frac{e}{8\pi} \]

\[ S_x = \frac{1}{2} \alpha e E_y \frac{m}{2\pi \hbar^2} \tau \]

V.M. Edelstein, Sol. State Commun. 73, 233 (1990)
**SPIN-ORBIT TORQUES**

**LLG EQUATION:**

\[
\frac{\partial M}{\partial t} = \gamma_e M \times (B_{\text{ext}} + B_{\text{exc}} + B_{\text{an}} + B_{\text{SO}}) + \frac{\lambda_{\text{eff}}}{M_s} M \times \frac{\partial M}{\partial t} - \frac{1}{(1 + \beta^2)M_s^2} M \times [M \times (u \cdot \nabla)M] - \frac{\beta}{(1 + \beta^2)M_s} M \times (u \cdot \nabla)M.
\]

\[u = (\mu_B P / e M_s) j\]

\[\gamma_e = -e / m_e\]

\[\beta = h / (J \tau_{\text{st}})\]

\[\lambda_{\text{eff}} = \lambda_G + \lambda_{\text{SP}}\]

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**SPIN TRANSFER TORQUE IN SPIN VALVES**

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**SPIN – ORBIT (RASHBA) TORQUE**

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**SPIN – HALL TORQUE**

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Sinova et al., Rev. Mod Phys. 87, 1213 (2015);
X. Fan et al., Nature Communications 4, 1799 (2013);
P. Gambardella, I. M. Miron, Phil. Trans. R. Soc. A, 369, 3175 (2011);
DETERMINATION OF THE SOTs

HARMONIC HALL VOLATGE MEASUREMENTS

- M. Hayashi, J. Kim, M. Yamanouchi, and H. Ohno, “Quantitative characterization of the spin-orbit torque using harmonic Hall voltage measurements”, PHYSICAL REVIEW B 89, 144425 (2014)

\[ E = -K_{\text{EFF}} \cos^2 \theta - K_I \sin^2 \varphi \sin^2 \theta - \tilde{M} \cdot \tilde{H} \]

\[
\begin{align*}
\tilde{M} &= M_S \hat{m}, \\
\hat{m} &= (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta), \\
\tilde{H} &= H (\sin \theta_H \cos \varphi_H, \sin \theta_H \sin \varphi_H, \cos \theta_H)
\end{align*}
\]

\[
\frac{\partial E}{\partial \theta} = 0, \quad \frac{\partial E}{\partial \varphi} = 0
\]

\[
\begin{align*}
\Delta \theta &= \frac{\partial \varphi}{\partial H_X} \Delta H_X + \frac{\partial \varphi}{\partial H_Y} \Delta H_Y + \frac{\partial \varphi}{\partial H_Z} \Delta H_Z \\
\Delta \varphi &= \frac{\partial \varphi}{\partial H_X} \Delta H_X + \frac{\partial \varphi}{\partial H_Y} \Delta H_Y + \frac{\partial \varphi}{\partial H_Z} \Delta H_Z
\end{align*}
\]

\[
\begin{align*}
\frac{\partial E}{\partial H_i} &= 0, \quad \frac{\partial \varphi}{\partial H_i} = 0 \\
\frac{\partial \varphi}{\partial H_i} &= 0
\end{align*}
\]

\[
\begin{align*}
\Delta \theta &= \cos \theta_0 (\Delta H_X \cos \varphi_H + \Delta H_Y \sin \varphi_H) - \sin \theta_0 \Delta H_Z \\
&= (H_K - H_A \sin^2 \varphi_H) \cos 2\theta_0 + H \cos (\theta_H - \theta_0)
\end{align*}
\]

\[
\begin{align*}
\Delta \varphi &= -\Delta H_X \sin \varphi_H + \Delta H_Y \cos \varphi_H - H_A \sin \theta_0 \cos 2\varphi_H + \sin \theta_H \\
&= K_{\text{EFF}} M_S^2, \quad H_A = \frac{2K_I}{M_S}
\end{align*}
\]

\[
|H_A| \ll |H \sin \theta_H|, \quad \varphi_0 = \varphi_H
\]

\[
\begin{align*}
R_{XY} &= \frac{1}{2} \Delta R_A \cos \theta + \frac{1}{2} \Delta R_P \sin^2 \theta \sin 2\varphi \\
\theta &= \theta_0 + \Delta \theta, \varphi = \varphi_0 + \Delta \varphi \\
R_{XY} &\approx \frac{1}{2} \Delta R_A (\cos \theta_0 - \Delta \theta \sin \theta_0) \\
&+ \frac{1}{2} \Delta R_P (\sin^2 \theta_0 + \Delta \theta \sin 2\theta_0)(\sin 2\varphi_0 + 2\Delta \varphi \cos 2\varphi_0) \Delta \varphi, \Delta \theta \ll 1
\end{align*}
\]

\[
\begin{align*}
V_{XY} &= R_{XY} I \\
I &= \Delta I \sin \omega t \\
V_{XY} &= V_0 + V_\omega \sin \omega t + V_{2\omega} \cos 2\omega t \\
V_0 &= \frac{1}{2} (B_\theta + B_\varphi) \Delta I, \\
V_\omega &= A \Delta I, \\
V_{2\omega} &= -\frac{1}{2} (B_\theta + B_\varphi) \Delta I.
\end{align*}
\]

\[
\begin{align*}
A &= \frac{1}{2} \Delta R_A \cos \theta_0 + \frac{1}{2} \Delta R_P \sin^2 \theta_0 \sin 2\varphi_0, \\
B_\theta &= -\frac{1}{2} (\Delta R_A \sin \theta_0 + \Delta R_P \sin 2\theta_0 \sin 2\varphi_0) \Delta \theta, \\
B_\varphi &= \Delta R_P \sin^2 \theta_0 \cos 2\varphi_0 \Delta \varphi.
\end{align*}
\]
DETERMINATION OF THE SOTs

HARMONIC HALL VOLTAGE MEASUREMENTS

- M. Hayashi, J. Kim, M. Yamanouchi, and H. Ohno, "Quantitative characterization of the spin-orbit torque using harmonic Hall voltage measurements", PHYSICAL REVIEW B 89, 144425 (2014)


\[
\begin{align*}
V_{XY} &= R_{XY} I \\
I &= \Delta I \sin \omega t \\

V_o &\approx \pm \frac{1}{2} R_A \left[ 1 - \frac{1}{2} \left( \frac{H \sin \theta_H}{H_K \pm H \cos \theta_H} \right)^2 \right] \Delta I \\
V_{2o} &\approx -\frac{1}{4} \left[ \pm R_A \left( \Delta H_X \cos \varphi_H + \Delta H_Y \sin \varphi_H \right) \\
&\quad + 2\Delta R_P \left( \Delta H_X \sin \varphi_H + \Delta H_Y \cos \varphi_H \right) \cos 2\varphi_H \right] \frac{H \sin \theta_H}{(H_K \pm H \cos \theta_H)^2} \Delta I \\
B &\equiv \left( \frac{\partial V_{2o}}{\partial H} \right) \frac{\partial^2 V_{2o}}{\partial H^2} \\
&= -\frac{1}{2} \left[ \left( \Delta H_X \pm \frac{2\Delta R_P}{\Delta R_A} \cos 2\varphi_H \Delta H_Y \right) \cos \varphi_H + \left( \Delta H_Y \pm \frac{2\Delta R_P}{\Delta R_A} \cos 2\varphi_H \Delta H_X \right) \sin \varphi_H \right] \\

\Delta H_X &= -2 \left( \frac{B_X \pm 2\xi B_Y}{1 - 4\xi^2} \right) \\
\Delta H_Y &= -2 \left( \frac{B_Y \pm 2\xi B_X}{1 - 4\xi^2} \right) \\

\xi &\equiv \frac{\Delta H_F}{\Delta R_A} \\
B_X &= \left( \frac{\partial H}{\partial \theta_H} \right)_{\varphi_H} |_{\theta_H} \\
B_Y &= \left( \frac{\partial H}{\partial \theta_H} \right)_{\varphi_H} |_{\theta_H} \\
\hat{\theta}_H &\equiv \left( \frac{\partial H}{\partial \theta_H} \right)_{\varphi_H} \\
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \hat{m}}{\partial t} &= -\gamma \hat{m} \times \left( -\frac{\partial E}{\partial \hat{m}} + \Delta \bar{H} \right) + \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t} \\
\frac{\partial \hat{m}}{\partial t} &= -\gamma \hat{m} \times \left( -\frac{\partial E}{\partial \hat{m}} + a_J (\hat{m} \times \hat{p}) + b_J \hat{p} \right) + \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t} \\
\end{align*}
\]

\( \hat{p} \) - represents the magnetization direction of the reference layer in the STT case or average spin direction of electrons diffusing into the magnetic layer in SOT case
PHENOMENOLOGICAL DESCRIPTION OF SOTs

**MOTIVATION:**

J. Kim, J. Sinha, S. Mitani, M. Hayashi, S. Takahashi, S. Maekawa, M. Yamanouchi, and H. Ohno,
*Anomalous temperature dependence of current-induced torques in CoFeB/MgO heterostructures with Ta-based underlayers*, PHYSICAL REVIEW B 89, 174424 (2014)

- **DRIFT-DIFFUSION FORMALISM**

\[
\Delta H_j = -\frac{\hbar}{2|e|t_F M_S} (\hat{m} \times \vec{Q}_z^{N|F})_j
\]

\[
\Delta H_x = -\theta_{SH} \frac{\hbar}{2|e| M_{STF}} \frac{J_N}{M_N} \left(1 - \frac{1}{\cosh(d/\lambda_N)}\right) \left[\frac{(1 + g_r) g_r + g_i^2}{(1 + g_r)^2 + g_i^2}\right] \text{sgn}(\hat{m} \times \hat{e}_y)_x,
\]

\[
\Delta H_y = -\theta_{SH} \frac{\hbar}{2|e| M_{STF}} \frac{J_N}{M_N} \left(1 - \frac{1}{\cosh(d/\lambda_N)}\right) \left[\frac{g_i}{(1 + g_r)^2 + g_i^2}\right]
\]

\[
g_r = \text{Re}[G_{\text{MIX}}] \rho_N \lambda_N \coth\left(\frac{d}{\lambda_N}\right)
\]

\[
g_i = \text{Im}[G_{\text{MIX}}] \rho_N \lambda_N \coth\left(\frac{d}{\lambda_N}\right)
\]

**BOUNDARY CONDITIONS:**

\[
Q_{iz}(z = 0) = 0,
\]

\[
Q_{iz}(z = d) = Q_{iz}^{N|F}
\]
MOTIVATION:

J. Kim, J. Sinha, S. Mitani, M. Hayashi, S. Takahashi, S. Maekawa, M. Yamanouchi, and H. Ohno,
*Anomalous temperature dependence of current-induced torques in CoFeB/MgO heterostructures with Ta-based underlayers*, PHYSICAL REVIEW B 89, 174424 (2014)

RESULTS

FIG. 6. (Color online) Underlayer thickness dependence of (a) and (b) $\Delta H_J/J_N$ and (c) and (d) $\Delta H_L/J_N$ for the TaN underlayer film. The measurement temperature is 100 K for (a) and (c) and RT ($\sim$295 K) for (b) and (d). Solid lines are calculated using Eqs. (7) and (8) with the following parameters: $M_S = 1200$ emu/cm$^3$, $\rho_N = 395$ $\mu$Ω-cm, $\lambda_S = 2.5$ nm, $\theta_{SH} = -0.08$, and the real and imaginary parts of the spin mixing conductance ($G_{MIX}$), which are shown by the solid symbols in Fig. 7(a).

FIG. 7. (Color online) (a) Temperature dependence of the real and imaginary parts of the spin mixing conductance ($G_{MIX}$) obtained by the fitting shown in Fig. 6. Linear fit to the data (symbols) is shown by the solid line. Temperature dependence of (b) $\Delta H_J/J_N$ and (c) $\Delta H_L/J_N$ for the TaN underlayer film with $d \sim 8.9$ nm. Solid lines are calculated using Eqs. (7) and (8) with the following parameters: $M_S = 1200$ emu/cm$^3$, $\rho_N = 395$ $\mu$Ω-cm, $\lambda_S = 2.5$ nm, $\theta_{SH} = -0.08$, and the interpolated real and imaginary parts of $G_{MIX}$ shown by the solid lines in (a).
PHENOMENOLOGICAL DESCRIPTION OF SOTs

EFFECTS ON INTERFACE

The NM layer and the FM layer affect the electronic structure of each other close to the interface:

- The magnetic proximity effect can induce a magnetic moment in the NM layer (the material with strong SOC).
- The spin-orbit proximity effect can induce spin-orbit effects in the FM layer.
- Lattice mismatch between materials: thickness fluctuations, formation of dislocation.
- ......

- **Spin backflow** (even from a „well-ordered” interface)
- **Spin memory loss**
  due to intermixing and disorder at the NM|FM interface.
PHENOMENOLOGICAL DESCRIPTION OF SOTs

**EFFECTS ON INTERFACE: TRILAYER SPIN DIFFUSION MODEL**

\[
\bar{j}^N_s(z) = -\frac{1}{2|e|\rho_N} \frac{\partial \bar{\mu}^N_s(z)}{\partial z} - \theta_{SH} J_N \hat{y} \\
\frac{\partial^2 \bar{\mu}^N_s(z)}{\partial z^2} = \frac{\bar{\mu}^N_s(z)}{\lambda_{s,N}^2} \\
\bar{\mu}^N_s(z) = \bar{C} \exp\left(-\frac{z}{\lambda_N}\right) + \bar{D} \exp\left(\frac{z}{\lambda_N}\right)
\]

\[
\bar{j}^I_s(z) = -\frac{1}{2|e|\rho_I} \frac{\partial \bar{\mu}^I_s(z)}{\partial z} \\
\frac{\partial^2 \bar{\mu}^I_s(z)}{\partial z^2} = \frac{\bar{\mu}^I_s(z)}{\lambda_{s,I}^2} \\
\bar{\mu}^I_s(z) = \bar{A} \exp\left(-\frac{z}{\lambda_I}\right) + \bar{B} \exp\left(\frac{z}{\lambda_I}\right)
\]

\[
|e|^\bar{j}^F_s = G_r \hat{m} \times \hat{m} \times \bar{\mu}^I_s(0) + G_i \hat{m} \times \bar{\mu}^I_s(0)
\]

**BOUNDARY CONDITIONS:**

\[
\bar{j}^N_s(z = d_I) = \bar{j}^I_s(z = d_I) \\
\bar{j}^N_s(z = 0) = |\bar{j}^F_s| \\
\bar{j}^N_s(z = d_I + d_N) = 0 \\
\bar{\mu}^I_s(z = d_I) = \bar{\mu}^N_s(z = d_I)
\]

\[
\bar{\mu}^I_s(0) = 2|e|\theta_{SH} J_N \hat{y} \left[ \frac{\tanh\left(\frac{d_N}{2\lambda_N}\right) \text{csch}\left(\frac{d_I}{\lambda_I}\right)}{\frac{1}{\rho_I \lambda_I} \coth\left(\frac{d_N}{\lambda_N}\right) + \frac{1}{\rho_N \lambda_N} \coth\left(\frac{d_I}{\lambda_I}\right)} \right] \\
+ 2|e|^\bar{j}^F_s \left[ \frac{\coth\left(\frac{d_N}{\lambda_N}\right) \coth\left(\frac{d_I}{\lambda_I}\right)}{\frac{1}{\rho_I \lambda_I} \coth\left(\frac{d_N}{\lambda_N}\right) + \frac{1}{\rho_N \lambda_N} \coth\left(\frac{d_I}{\lambda_I}\right)} \right]
\]
PHENOMENOLOGICAL DESCRIPTION OF SOTs

**EFFECTS ON INTERFACE:** TRILAYER SPIN DIFFUSION MODEL

**DAMPING-LIKE AND FIELD-LIKE TORQUE EFFICIENCY:**

\[
\xi_{DL} = \frac{2|e|\mu_0 M_{STF}}{\hbar J_N} \Delta H_L \\
\xi_{FL} = \frac{2|e|\mu_0 M_{STF}}{\hbar J_N} \Delta H_T
\]

\[
\xi_{DL} = -\theta_{SH} \frac{\tanh(\frac{d_N}{2\lambda_N}) \text{csch}(\frac{d_L}{\lambda_I})}{\coth(\frac{d_N}{\lambda_N}) \coth(\frac{d_L}{\lambda_I}) + \frac{\rho_I}{\rho_N} \frac{\lambda_I}{\lambda_N}} \frac{g_r}{(1 + g_r)^2 + g_i^2} m_z
\]

\[
\xi_{FL} = -\theta_{SH} \frac{\tanh(\frac{d_N}{2\lambda_N}) \text{csch}(\frac{d_L}{\lambda_I})}{\coth(\frac{d_N}{\lambda_N}) \coth(\frac{d_L}{\lambda_I}) + \frac{\rho_I}{\rho_N} \frac{\lambda_I}{\lambda_N}} \frac{g_i}{(1 + g_r)^2 + g_i^2}
\]

**WITHOUT INTERFACE LAYER**

\[
\xi_{DL} = -\theta_{SH} \left(1 - \frac{1}{\cosh(\frac{d}{\lambda_N})}\right) \frac{(1 + g_r)g_r + g_i^2}{(1 + g_r)^2 + g_i^2}
\]

\[
\xi_{FL} = -\theta_{SH} \left(1 - \frac{1}{\cosh(\frac{d}{\lambda_N})}\right) \frac{g_i}{(1 + g_r)^2 + g_i^2}
\]

\[
g_{r,i} = 2G_{r,i} \frac{\coth(\frac{d_N}{\lambda_N}) \coth(\frac{d_L}{\lambda_I}) + \frac{\rho_I}{\rho_N} \frac{\lambda_I}{\lambda_N}}{\frac{1}{\rho_I} \coth(\frac{d_N}{\lambda_N}) + \frac{1}{\rho_N} \coth(\frac{d_L}{\lambda_I})}
\]

\[G_i = \text{Im}\{G_{\text{mix}}\}, \ G_r = \text{Re}\{G_{\text{mix}}\}\]

\[g_{r,i} = G_{r,i} \rho_N \lambda_N \coth(d/\lambda_N)\]
PHENOMENOLOGICAL DESCRIPTION OF SOTs

**EFFECTS ON INTERFACE: TRILAYER SPIN DIFFUSION MODEL – RESULTS**

![Graphs showing spin-torque efficiencies](image)

**FIG. 6:** Spin-torque efficiencies with theoretical fit for the case of additional and no interfacial layer in the model for indicated values of spin Hall angle $\theta_{SH}$ and indicated values of Ta underlayer thickness for Ta spin diffusion length $\lambda_N = 2.5$ nm.
TRILAYER SPIN DIFFUSION MODEL: SUMMARY

• $\text{Im}[G_{\text{MIX}}]$ comparable in size to its real counterpart
  ✓ large angle rotation of the spin direction of spin current
  ✓ presence of spin Hall torque

• $\text{Negative Re}[G_{\text{MIX}}]$ !!!
  ✓ model failure
  ✓ poor band matching between NM and the FM layers
  ✓ flow of spin current from the FM layer to the NM layer

AHE or SHE in FM layer should be checked!
ANOMALOUS HALL EFFECT IN Ta|CoFeB

The anomalous Hall effect in the perpendicular Ta/CoFeB/MgO thin films

S. B. Wu, T. Zhu, X. F. Yang, and S. Chen

1Institute of Physics and Beijing National Laboratory for Condensed Matter Physics, Chinese Academy of Sciences, Beijing 100190, People’s Republic of China
2Department of Electronic Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, People’s Republic of China

(Presented 17 January 2013; received 2 November 2012; accepted 20 December 2012; published online 25 March 2013)

The anomalous Hall effect (AHE) in the perpendicular Ta/CoFeB/MgO thin film has been investigated. Between the AHE coefficient ($R_S$) and longitudinal resistivity ($\rho_{xx}$), a linear behavior of $R_S/\rho_{xx}$ versus $\rho_{xx}$ can be found. Moreover, the conductivity of the film is about $5 \times 10^4$ S/cm, which suggests that AHE in the Ta/CoFeB/MgO film be dominated by the intrinsic or scattering-independent mechanism. © 2013 American Institute of Physics. [http://dx.doi.org/10.1063/1.4796192]

Giant linear anomalous Hall effect in the perpendicular CoFeB thin films

T. Zhu, P. Chen, Q. H. Zhang, R. C. Yu, and B. G. Liu

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Giant anomalous Hall effect in a perpendicular CoFeB thin film has been reported, which satisfies large anomalous Hall resistivity and low switching field at same time. The largest sensitivity of linear anomalous Hall effect reaches 2376 $\Omega$/kOe, which is 21 times larger than that of the best semiconductor. Our results suggest the low cost MgO/CoFeB/Ta thin film can be a potential candidate for highly sensitive magnetic field detecting. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4878538]
TRILAYER SPIN DIFFUSION MODEL: SUMMARY

- Im[$G_{\text{Mix}}$] comparable in size to its real counterpart
  - large angle rotation of the spin direction of spin current
  - presence of spin Hall torque

- Negative Re[$G_{\text{Mix}}$] !!!
  - model failure
  - poor band matching between NM and the FM layers
  - flow of spin current from the FM layer to the NM layer

FURTHER MODIFICATION OF TRILAYER SPIN DIFFUSION MODEL

\[
\bar{J}_N^s(z) &= -\frac{1}{2|e|\rho_N} \frac{\partial \bar{\mu}_N^s(z)}{\partial z} - \theta_{SH} J_N \hat{y} \\
\frac{\partial^2 \bar{\mu}_N^s(z)}{\partial z^2} &= \frac{\bar{\mu}_N^s(z)}{\lambda_{s,N}^2} \\
\bar{\mu}_N^s(z) &= \bar{C} \exp(-\frac{z}{\lambda_N}) + \bar{D} \exp(\frac{z}{\lambda_N})
\]

\[
\bar{J}_I^s(z) &= -\frac{1}{2|e|\rho_I} \frac{\partial \bar{\mu}_I^s(z)}{\partial z} \\
\frac{\partial^2 \bar{\mu}_I^s(z)}{\partial z^2} &= \frac{\bar{\mu}_I^s(z)}{\lambda_{s,I}^2} \\
\bar{\mu}_I^s(z) &= \bar{A} \exp(-\frac{z}{\lambda_I}) + \bar{B} \exp(\frac{z}{\lambda_I})
\]

\[
|e|\bar{\gamma}_s^F = G_r \hat{m} \times \hat{m} \times \bar{\mu}_s^I(0) + G_i \hat{m} \times \bar{\mu}_s^I(0)
\]
Spin Transport at Interfaces with Spin-Orbit Coupling: Phenomenology

V. P. Amin\(^1,2,*\) and M. D. Stiles\(^2\)

\[
\begin{align*}
j_s(z) &= \frac{\sigma_{FM}^{\text{FM}}}{e} \frac{\partial \mu_c(z)}{\partial z} - \frac{\sigma_{FM}^{\text{FM}}}{e} \frac{\partial \mu_s(z)}{\partial z} \\
\frac{1}{eN_{FM}^{\text{FM}}} \frac{\partial j_s(z)}{\partial z} &= -\frac{1}{\tau_{\text{sf}}^{\text{FM}}} \mu_s(z) - \frac{1}{\tau_{\text{ex}}} \mu_s(z) \times \hat{m} - \frac{1}{\tau_{\text{dp}}} \hat{m} \times \mu_s(z) \times \hat{m} \\
j_s^\text{H}(z) &= \frac{\sigma_{HM}^{\text{HM}}}{e} \frac{\partial \mu_s(z)}{\partial z} + j_s^\text{H} \\
\frac{1}{eN_{HM}^{\text{HM}}} \frac{\partial j_s^\text{H}(z)}{\partial z} &= -\frac{1}{\tau_{\text{sf}}^{\text{HM}}} \mu_s(z).
\end{align*}
\]

**BOUNDARY CONDITIONS:**

- \(j_{\perp}(0^-) = G_R \mu_{\perp}(0^-) + j_{\perp}^E(0^-) = \sigma(\hat{m})E\)
- \(j_{\perp}(0^+) = \Gamma_{FM} \mu_{\perp}(0^+) + j_{\perp}^E(0^+) = \gamma_{FM}(\hat{m})E\)
- \(\tau_{\text{mag}} = (G_R - \Gamma_{FM}) \mu_{\perp}(0^-) + \tau^E = \gamma_{\text{mag}}(\hat{m})E\)

**THE RASHBA MODEL OF SOC**

- conductance matrix \(- G_R = \begin{bmatrix} \text{Re}[G_{11}] & -\text{Im}[G_{11}] \\ \text{Im}[G_{11}] & \text{Re}[G_{11}] \end{bmatrix}\)
- torkance tensor \(- \Gamma_{FM}\)
- conductivity tensor \(- \sigma(\hat{m})\)
- torkivity tensors \(- \gamma_{\text{mag}}(\hat{m})\)
- \(\gamma_{FM}(\hat{m})\)

The torque on magnetization at the interface:

\[V(r) = \frac{k_F}{m} \delta(z) \left(u_0 + u_{ex} \sigma \cdot \hat{m} + u_R \sigma \cdot (\hat{k} \times \hat{z})\right)\]
Spin Pumping in the Presence of Spin-Orbit Coupling

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Spin pumping and related phenomena have been observed recently in heavy metals and topological insulators, where the spin-orbit coupling plays an essential role. We have developed a spin-pumping formalism that explicitly includes the spin-orbit coupling at interfaces and disorder in the layers. Spin pumping across an interface with spin-orbit coupling and the attendant backflow are treated on an equal footing. We resolve some long-standing issues on the conflicting conclusions about the spin-diffusion length for Pt, and the origin of spin-memory loss at interfaces with heavy metals. In addition, we predict some heretofore unanticipated spin-pumping phenomena.

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- spin pumping conductivity is given by retarded and advanced Green functions

$$j^s = \frac{\hbar}{4\pi} \left( g_s \mathbf{m} \times \frac{d\mathbf{m}}{dt} - g_i \frac{d\mathbf{m}}{dt} \right)$$

$$j_i(r, t) = \frac{\hbar}{4\pi} \left[ \Gamma_i^{re}(r) \mathbf{m} \times \frac{d\mathbf{m}}{dt} - \Gamma_i^{im}(r) \frac{d\mathbf{m}}{dt} \right]$$

$$\Gamma_i(r) = \Gamma_i^{re}(r) + i\Gamma_i^{im}(r) \quad \Gamma_i(r) = \frac{J_{ex}}{m_e} \frac{\hbar^2}{\int_{\text{FM}} d^3r' g_i^R(r, r') \bar{\partial}_i g_i^A(r', r)}$$

- disoredrs included explicitly
- SOC at the interface
- backflow current
Spin Transport at Interfaces with Spin-Orbit Coupling: Phenomenology

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(Dated: April 25, 2016)

Spin transport remains poorly understood in multilayer systems with interfacial spin-orbit coupling. While the important consequences of interfacial spin-orbit coupling can be captured by a spin-dependent Boltzmann equation, currently they cannot be captured by drift-diffusion models, which are the primary tools used for analyzing experiments. Here we present boundary conditions suitable for drift-diffusion models that capture the phenomenology of spin-orbit torques at interfaces. We compare solutions of the drift-diffusion equations using these boundary conditions to solutions of the spin-dependent Boltzmann equation for a heavy metal/ferromagnet bilayer. We find that the drift-diffusion equations predict spin torques in quantitative agreement with the Boltzmann equation and allow for a much simpler interpretation of the results. A key feature of these boundary conditions is their ability to capture the scattering that carriers experience while flowing along an interface. In the presence of interfacial spin-orbit coupling, this scattering generates spin currents that flow away from the interface. In heavy metal/ferromagnet bilayers, these spin currents provide an important mechanism for the creation of damping-like and field-like torques; they also lead to possible reinterpretations of experiments in which interfacial contributions to spin torques are thought to be suppressed.

\[
\dot{j}_s(z) = \frac{\sigma_{FM}}{e} \vec{P} \cdot \vec{m} - \frac{\sigma_{FM}}{e} \frac{\partial \mu_c(z)}{\partial z} - \frac{\sigma_{FM}}{e} \frac{\partial \mu_s(z)}{\partial z} = \frac{1}{e N_{sFM}} \frac{\partial \mu_s(z)}{\partial z} - \frac{1}{\tau_{sFM}} \mu_s(z) - \frac{1}{\tau_{ex}} \frac{\partial \mu_s(z)}{\partial z} - \frac{1}{\tau_{dp}} \mu_s(z) \times \vec{m} - \frac{1}{\tau_{dp}} \mu_s(z) \times \vec{m}
\]

\[
\dot{j}_s(z) = -\frac{\sigma_{HM}}{e} \frac{\partial \mu_s(z)}{\partial z} + j_{sH} \sigma_{sH} E \times \hat{z}
\]

\[
\dot{j}_s(z) = -\frac{1}{e N_{sHM}} \frac{\partial \mu_s(z)}{\partial z} = -\frac{1}{\tau_{sHM}} \mu_s(z).
\]

\[
\dot{j}_s(0^-) = G_R \mu_s(0^-) + j_{sH}^E(0^-) = \sigma(\vec{m}) \dot{E}
\]

\[
\dot{j}_s(0^+) = \Gamma_{FM} \mu_s(0^+) + j_{sH}^E(0^+) = \gamma_{FM} \dot{E}
\]

\[
\tau_{mag}^m = (G_R - \Gamma_{FM}) \mu_s(0^-) + \tau_{mag}^E \gamma_{mag}(\vec{m}) \dot{E}
\]

\[
G_R = \begin{pmatrix} Re[G_{11}] & -Im[G_{11}] \\ Im[G_{11}] & Re[G_{11}] \end{pmatrix}
\]

\[
V(\vec{r}) = \frac{\hbar^2 k_F^2}{m} \delta(z) \left( u_0 + u_{ex} \sigma \cdot \vec{m} + u_R \sigma \cdot (\hat{k} \times \hat{z}) \right)
\]