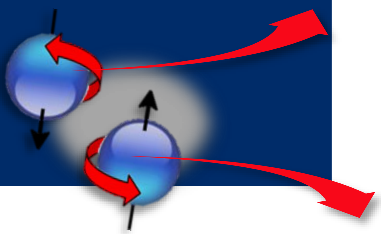
THEORY OF SPIN HALL EFFECT AND INTERFACE SPIN-ORBIT COUPLING



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NANOSPIN: Nanoscale spin torque devices for spin electronics

Summarizing MEETING, 11th-12th of July 2016

SWISS CONTRIBUTION

Polish-Swiss Research Programme

OUTLINE

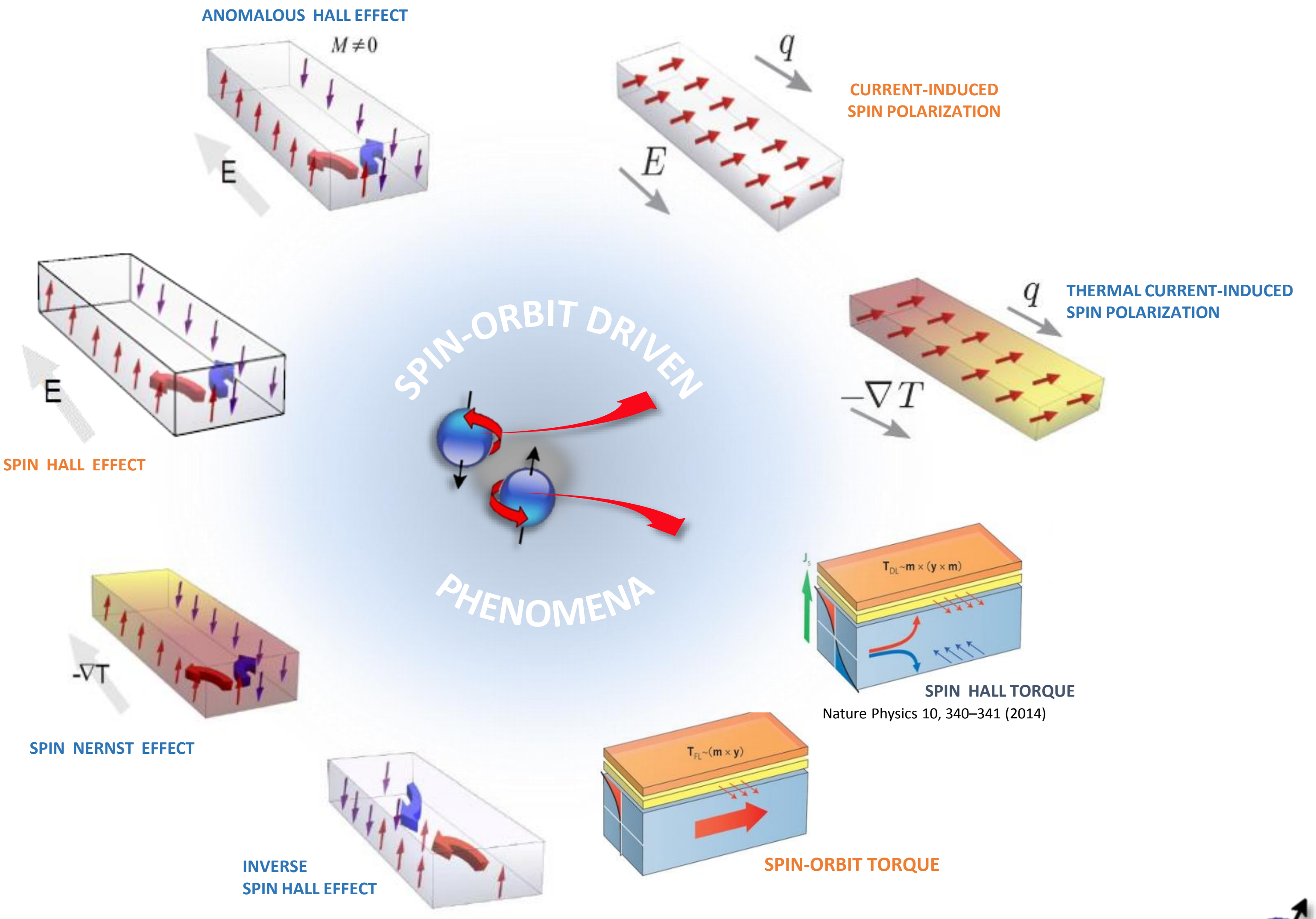
- INTRODUCTION: SPIN ORBIT DRIVEN PHENOMENA
  - MICROSCOPIC MECHANISMS LEADING TO AHE/SHE
  - SPIN TORQUES DUE TO SPIN-ORBIT COUPLING
  - SHE AND CISP IN SEMICONDUCTOR HETEROSTRUCTURES WITH RASHBA SOI

- DETERMINATION OF SPIN-ORBIT TORQUES:

  THEORETICAL DESCRIPTION OF HARMONIC HALL VOLTAGE MEASUREMENTS

- PHENOMENOLOGICAL DESCRIPTION OF SOTs
  - DRIFT-DIFFUSION MODEL FOR Ta|CoFeB
    - literature – model for Ta|CoFeB by Kim et.al.
    - results for trilayer spin-diffusion model of Ta|interface|CoFeB
  - TRILAYER SPIN-DIFFUSION MODEL: SUMMARY
  - AHE IN Ta|CoFeB
  - ANOTHER APPROACHES

SPIN-ORBIT DRIVEN PHENOMENA



MICROSCOPIC MECHANISMS

▪ INTRINSIC CONTRIBUTION

BAND STRUCTURE TOPOLOGY

An external electric field leads to an interband coherence that gives rise to contribution to the transverse velocity – an anomalous velocity related to Berry phase curvature

$$v_y = \frac{\partial \varepsilon_{n\mathbf{k}}}{\partial k_y} - eE_x \mathcal{F}_z^n$$

anomalous velocity

$$\hat{H}_{SO} = -\frac{1}{4m^2c^2}\hat{\sigma}\cdot(\mathrm{p}\times\nabla V)$$

$$\hat{H}_E = e\mathbf{E}\cdot\hat{\mathbf{r}} \qquad \langle u_{n\mathbf{k}}|\hat{H}_E|u_{\bar{n}\mathbf{k}}\rangle = ie\mathbf{E}\cdot\langle u_{n\mathbf{k}}|\frac{\partial u_{\bar{n}\mathbf{k}}}{\partial \mathbf{k}}\rangle \neq 0 \qquad \bar{n}\neq n$$

$$\hat{H}_0 \longrightarrow |u_{n\mathbf{k}}\rangle$$

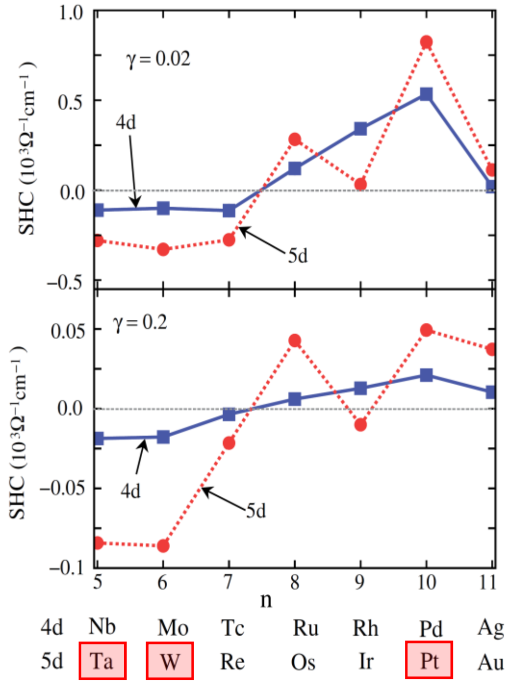
$$\hat{H} = \hat{H}_0 + eE_x\hat{x} \longrightarrow |u'_{n\mathbf{k}}\rangle = |u_{n\mathbf{k}}\rangle + ieE_x\sum_{\bar{n}\neq n}\frac{\langle u_{\bar{n}\mathbf{k}}|\frac{\partial u_{n\mathbf{k}}}{\partial k_x}\rangle}{\epsilon_{n\mathbf{k}}-\epsilon_{\bar{n}\mathbf{k}}}|u_{\bar{n}\mathbf{k}}\rangle$$

$$\Psi'_{n\mathbf{k_c}}(\mathrm{r},t) = e^{-i\hat{H}t}\Psi'_{n\mathbf{k_c}}(\mathrm{r})$$

$$\Psi'_{n\mathbf{k_c}}(\mathrm{r}) = \int\frac{d^D\mathbf{k}}{L^{D/2}}a(\mathbf{k})exp\left\{i\left(\mathbf{k}\cdot\mathbf{r}\right)\right\}|u'_{n\mathbf{k}}\rangle$$

$$v_y = \frac{d}{dt}\langle\Psi'_{n\mathbf{k_c}}(\mathbf{r},t)|\hat{y}|\Psi'_{n\mathbf{k_c}}(\mathbf{r},t)\rangle = \langle u'_{n\mathbf{k}}| - i\left[i\frac{\partial}{\partial k_y}, H_0\right]|u'_{n\mathbf{k}}\rangle$$

$$= \frac{\partial\epsilon_{n\mathbf{k}}}{\partial k_y} + ieE_x\left(\langle\frac{\partial u_{n\mathbf{k}}}{\partial k_y}|\frac{\partial u_{n\mathbf{k}}}{\partial k_x}\rangle - \langle\frac{\partial u_{n\mathbf{k}}}{\partial k_x}|\frac{\partial u_{n\mathbf{k}}}{\partial k_y}\rangle\right)$$



Intrinsic spin Hall conductivity for 4d and 5d transition metals (microscopic tight-binding calculations);
γ – a quasiparticle damping rate

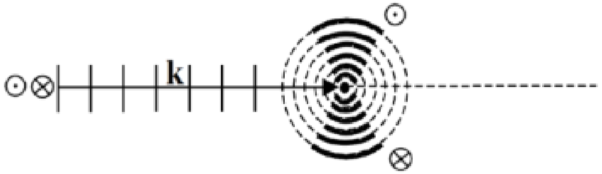
Tanaka, T., H. Kontani, M. Naito, T. Naito, D. Hirashima, K. Yamada, and J. Inoue, 2008, Phys. Rev. B 77, 165117.

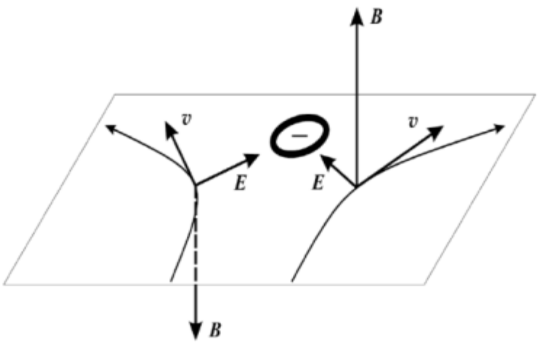
J. Sinova et al., Rev. Mod Phys. 87, 1213 (2015); G. Vignale, J. Supercond. Novel Magn. 23, 3 (2010); J. Schliemann, Int. J. Mod. Phys. B 20, 1015 (2006); M. I. Dyakonov and A. V. Khaetskii, in *Spin Physics in Semiconductors* edited by M. I. Dyakonov, Springer-Verlag, Berlin, Heidelberg, 2008, Chap. 8;

MICROSCOPIC MECHANISMS

- **EXTRINSIC CONTRIBUTION**

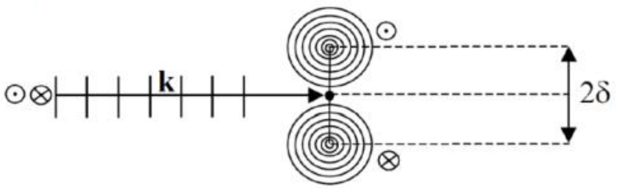
✓ SKEW SCATTERING





✓ SIDE JUMP

- EXTRINSIC SIDE-JUMP: The contribution arising from the non-spin-orbit coupled part of the wave-packet scattering off the spin-orbit coupled disordered.



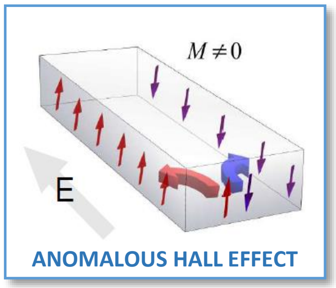
- INTRINSIC SIDE-JUMP: The contribution arising from the spin-orbit coupled part of the wave-packet formed by the Bloch electrons scattering off the scalar potential alone without spin-orbit coupling.

J. Sinova et al., Rev. Mod Phys. 87, 1213 (2015); G. Vignale, J. Supercond. Novel Magn. 23, 3 (2010);
J. Schliemann, Int. J. Mod. Phys. B 20, 1015 (2006); A. Crépieux, P. Bruno, Phys. Rev. B 64, 014416 (2001);
M. I. Dyakonov and A. V. Khaetskii, in *Spin Physics in Semiconductors* edited by M. I. Dyakonov, Springer-Verlag, Berlin, Heidelberg, 2008, Chap. 8;

MICROSCOPIC MECHANISMS: EXPERIMENTAL DATA vs THEORY

$$\sigma_{xy} = \sigma_{xy}^{\text{int}} + \sigma_{xy}^{\text{sj}} + \sigma_{xy}^{\text{ss}}$$

$\sim \tau^0$ $(\sim \sigma_{xx}^0)$ for $\sigma_{xy}^{\text{int}} + \sigma_{xy}^{\text{sj}}$; $\sim \tau^1$ $(\sim \sigma_{xx}^1)$ for σ_{xy}^{ss}



HIGH CONDUCTIVITY REGIME

$\sigma_{xx} > 0.5 \times 10^6\ (\Omega\ \text{cm})^{-1}$

$\sigma_{xy}^{AH} \sim \sigma_{xx}$

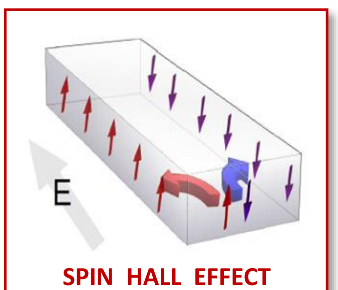
skew-scattering mechanism dominate

GOOD METAL REGIME

$\sigma_{xx} \sim 10^4 - 10^6\ (\Omega\ \text{cm})^{-1}$

$\sigma_{xy}^{AH} \sim \sigma_{xx}^0$

scattering-independent mechanisms or side jump dominate



BAD-METAL-HOPPING REGIME

$\sigma_{xx} < 10^4\ (\Omega\ \text{cm})^{-1}$

$\sigma_{xy}^{AH} \sim \sigma_{xx}^{1.6-1.8}$

there is currently no theory which predicts the observed scaling

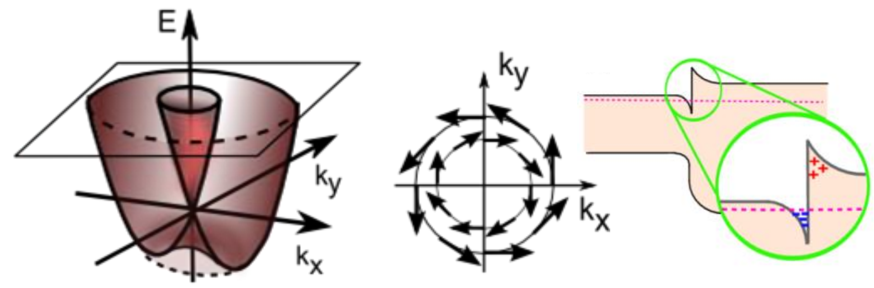
N. Nagaosa et al., Rev. Mod. Phys. 82, 1539 (2010); J. Sinova et al., Rev. Mod Phys. 87, 1213 (2015)

CISP IN 2D ELECTRON GAS WITH RASHBA SOI

$$H = \sum_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} H_{\mathbf{k}} \psi_{\mathbf{k}}$$

$$H_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m} + H_{RSO}$$

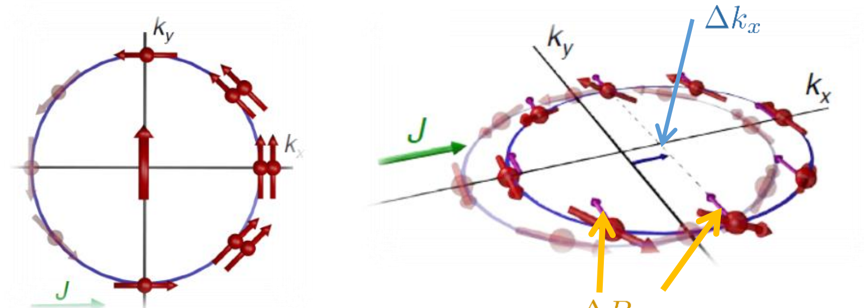
$$H_{RSO} = \alpha(k_y\sigma_x - k_x\sigma_y)$$



$$H_{SO} = \frac{\hbar}{2}\vec{B}_{eff}(\vec{k}) \cdot \vec{\sigma}$$

$$\vec{B}_{eff}(\vec{k}) = \frac{2\alpha}{\hbar}(k_y, -k_x, 0)$$

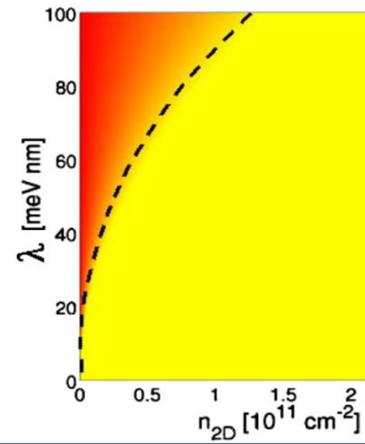
$$\vec{B}_{eff}(\vec{k}) \sim -\vec{k} \times \vec{E} \qquad \vec{E} = -\frac{1}{e}\nabla V(\vec{r})$$



J. Sinova et al., Rev. Mod. Phys. 87, 1213 (2015)

$$\mu > 0 :$$

$$\sigma^{s_z}_{xy} = \frac{e}{8\pi} \qquad S_x = \kappa\frac{1}{2}\alpha e E_y \frac{m}{2\pi\hbar^2}\tau$$



J. Sinova et al., Phys. Rev. Lett. 92, 126603 (2004)
N. A. Sinitsyn et al., Phys. Rev. B 70, 081312(R) (2004)
V.M. Edelstein, Sol. State Communs. 73, 233 (1990)
C. Gorini et al., Phys. Rv. B **78**, 125327 (2008)
P. Schwab et al., Phys. Rev. B, Europhysics Lett. **90**, 67004 (2010)
L. Golub et al., Phys. Rev. B 84, 115303 (2011)
A.Dyrdal et al., Phys. Rev. B 87, 245309 (2013)

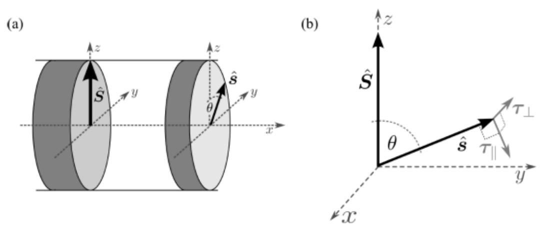
SPIN-ORBIT TORQUES

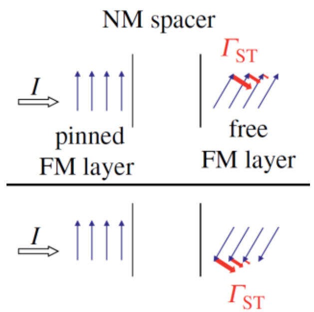
LLG EQUATION:

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma_e \mathbf{M} \times (\mathbf{B}_{ext} + \mathbf{B}_{exc} + \mathbf{B}_{an} + \mathbf{B}_{SO}) + \frac{\lambda_{eff}}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$$
$$- \frac{1}{(1+\beta^2) M_s^2} \mathbf{M} \times [\mathbf{M} \times (\mathbf{u} \cdot \nabla) \mathbf{M}] - \frac{\beta}{(1+\beta^2) M_s} \mathbf{M} \times (\mathbf{u} \cdot \nabla) \mathbf{M}.$$

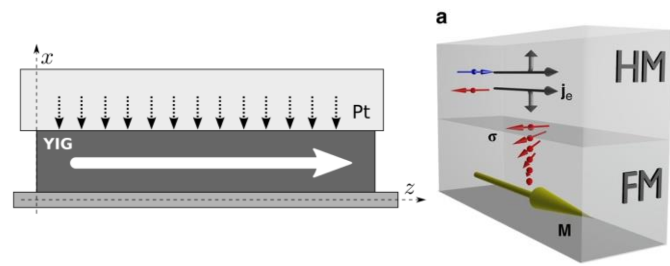
$$\mathbf{u} = (\mu_B P / e M_s) \mathbf{j}$$
$$\gamma_e = -e/m_e$$
$$\beta = \hbar / (J \tau_{sf})$$
$$\lambda_{eff} = \lambda_G + \lambda_{SP}$$

- SPIN TRANSFER TORQUE IN SPIN VALVES

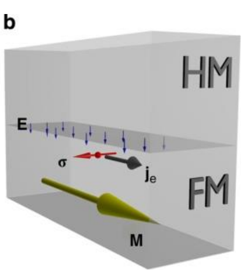


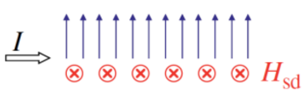


- SPIN – HALL TORQUE



- SPIN – ORBIT (RASHBA) TORQE

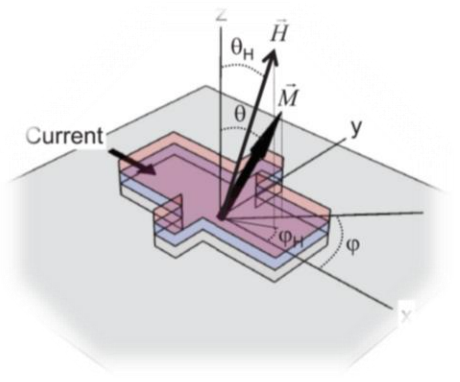




. Sinova et al., Rev. Mod Phys. 87, 1213 (2015);
X. Fan et al., Nature Communications 4, 1799 (2013);
P. Gambardella, I. M. Miron, Phil. Trans. R. Soc. A, 369, 3175 (2011);

DETERMINATION OF THE SOTs

HARMONIC HALL VOLATGE MEASUREMENTS



- M. Hayashi, J. Kim, M. Yamanouchi, and H. Ohno,
"Quantitative characterization of the spin-orbit torque using harmonic Hall voltage measurements",
PHYSICAL REVIEW B **89**, 144425 (2014)
- J. Kim, J. Sinha, M. Hayashi, M. Yamanouchi, S. Fukami, T. Suzuki, S. Mitani and H. Ohno,
" Layer thickness dependence of the current-induced effective field vector in Ta |CoFeB |MgO",
NATURE MATERIALS **12**, 240 (2013)

$$E = -K_{EFF}\cos^2\theta - K_I \sin^2\varphi \sin^2\theta - \vec{M}\cdot\vec{H}$$

$$\vec{M} = M_S\hat{m}, \quad \hat{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

$$\vec{H} = H(\sin\theta_H\cos\varphi_H, \sin\theta_H\sin\varphi_H, \cos\theta_H)$$

$$\frac{\partial E}{\partial\theta} = 0, \quad \frac{\partial E}{\partial\varphi} = 0 \quad \Rightarrow \quad (\theta_0, \varphi_0)$$

$$\Delta\theta = \frac{\partial\theta}{\partial H_X}\Delta H_X + \frac{\partial\theta}{\partial H_Y}\Delta H_Y + \frac{\partial\theta}{\partial H_Z}\Delta H_Z$$

$$\Delta\varphi = \frac{\partial\varphi}{\partial H_X}\Delta H_X + \frac{\partial\varphi}{\partial H_Y}\Delta H_Y + \frac{\partial\varphi}{\partial H_Z}\Delta H_Z$$

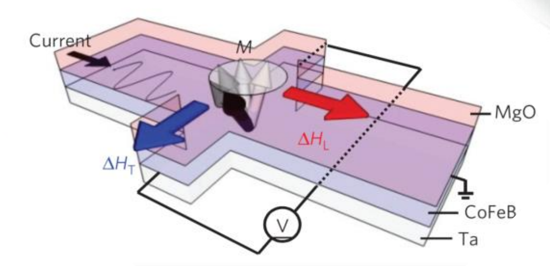
$$\frac{\partial}{\partial H_i}\left(\frac{\partial E}{\partial\theta}\right) = 0, \quad \frac{\partial}{\partial H_i}\left(\frac{\partial E}{\partial\varphi}\right) = 0 \quad \Rightarrow \quad \frac{\partial\theta}{\partial H_i}, \frac{\partial\varphi}{\partial H_i}$$

$$\Delta\theta = \frac{\cos\theta_0(\Delta H_X\cos\varphi_H + \Delta H_Y\sin\varphi_H) - \sin\theta_0\Delta H_Z}{(H_K - H_A\sin^2\varphi_H)\cos 2\theta_0 + H\cos(\theta_H - \theta_0)}$$

$$\Delta\varphi = \frac{-\Delta H_X\sin\varphi_H + \Delta H_Y\cos\varphi_H}{-H_A\sin\theta_0\cos 2\varphi_H + H\sin\theta_H}$$

$$H_K \equiv \frac{2K_{EFF}}{M_S}, \quad H_A \equiv \frac{2K_I}{M_S}$$

$$|H_A| \ll |H\sin\theta_H|, \ \varphi_0 = \varphi_H$$



$$R_{XY} = \tfrac{1}{2}\Delta R_A\cos\theta + \tfrac{1}{2}\Delta R_P\sin^2\theta\sin 2\varphi$$

$$\theta = \theta_0 + \Delta\theta, \varphi = \varphi_0 + \Delta\varphi$$

$$R_{XY} \approx \tfrac{1}{2}\Delta R_A(\cos\theta_0 - \Delta\theta\sin\theta_0) + \tfrac{1}{2}\Delta R_P(\sin^2\theta_0 + \Delta\theta\sin 2\theta_0)(\sin 2\varphi_0 + 2\Delta\varphi\cos 2\varphi_0) \quad \Delta\varphi, \Delta\theta \ll 1$$

$$V_{XY} = R_{XY} I, \quad I = \Delta I\sin\omega t$$

$$V_{XY} = V_0 + V_\omega\sin\omega t + V_{2\omega}\cos 2\omega t,$$
$$V_0 = \tfrac{1}{2}(B_\theta + B_\varphi)\Delta I,$$
$$V_\omega = A\Delta I,$$
$$V_{2\omega} = -\tfrac{1}{2}(B_\theta + B_\varphi)\Delta I,$$

$$A = \tfrac{1}{2}\Delta R_A\cos\theta_0 + \tfrac{1}{2}\Delta R_P\sin^2\theta_0\sin 2\varphi_0,$$
$$B_\theta = \tfrac{1}{2}(-\Delta R_A\sin\theta_0 + \Delta R_P\sin 2\theta_0\sin 2\varphi_0)\Delta\theta,$$
$$B_\varphi = \Delta R_P\sin^2\theta_0\cos 2\varphi_0\Delta\varphi.$$

DETERMINATION OF THE SOTs

❑ HARMONIC HALL VOLATGE MEASUREMENTS

- M. Hayashi, J. Kim, M. Yamanouchi, and H. Ohno,
”Quantitative characterization of the spin-orbit torque using harmonic Hall voltage measurements”,
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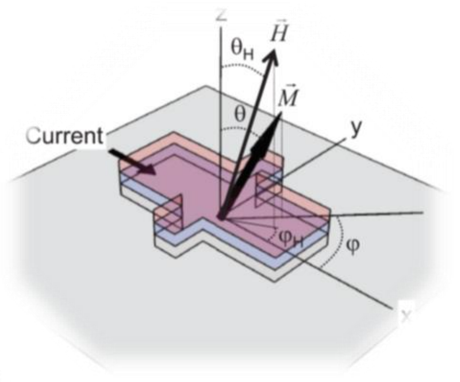
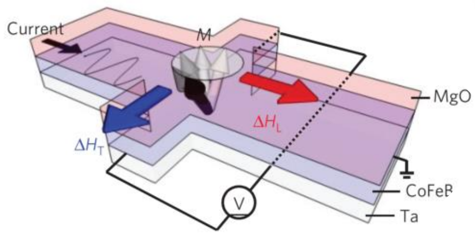
NATURE MATERIALS **12**, 240 (2013)





Out-of-plane magnetization

$$V_{XY} = R_{XY} I \qquad I = \Delta I \sin \omega t$$

$$V_{XY} = V_0 + V_\omega \sin \omega t + V_{2\omega} \cos 2\omega t,$$

$$V_\omega \approx \pm \frac{1}{2} \Delta R_A \left[1 - \frac{1}{2} \left(\frac{H \sin \theta_H}{H_K \pm H \cos \theta_H} \right)^2 \right] \Delta I$$

$$V_{2\omega} \approx -\frac{1}{4} [\mp \Delta R_A (\Delta H_X \cos \varphi_H + \Delta H_Y \sin \varphi_H) + 2\Delta R_P (-\Delta H_X \sin \varphi_H + \Delta H_Y \cos \varphi_H) \cos 2\varphi_H] \frac{H \sin \theta_H}{(H_K \pm H \cos \theta_H)^2} \Delta I$$

$$B \equiv \left(\frac{\partial V_{2\omega}}{\partial H} \Big/ \frac{\partial^2 V_\omega}{\partial H^2} \right) = -\frac{1}{2} \left[\left(\Delta H_X \mp 2 \frac{\Delta R_P}{\Delta R_A} \cos 2\varphi_H \Delta H_Y \right) \cos \varphi_H + \left(\Delta H_Y \pm 2 \frac{\Delta R_P}{\Delta R_A} \cos 2\varphi_H \Delta H_X \right) \sin \varphi_H \right]$$

$$\Delta H_X = -2 \frac{(B_X \pm 2\xi B_Y)}{1 - 4\xi^2} \qquad \Delta H_Y = -2 \frac{(B_Y \pm 2\xi B_X)}{1 - 4\xi^2}$$

$$\xi \equiv \frac{\Delta R_P}{\Delta R_A} \qquad B_X \equiv \left(\frac{\partial V_{2\omega}}{\partial H} / \frac{\partial^2 V_\omega}{\partial H^2} \right) \Big|_{\vec{H} \parallel \hat{x}} \qquad B_Y \equiv \left(\frac{\partial V_{2\omega}}{\partial H} / \frac{\partial^2 V_\omega}{\partial H^2} \right) \Big|_{\vec{H} \parallel \hat{y}}$$

LANDAU-LIFSHITZ-GILBERT (LLG) EQUATION

$$\frac{\partial \hat{m}}{\partial t} = -\gamma \hat{m} \times \left(-\frac{\partial E}{\partial \vec{M}} + \Delta \vec{H} \right) + \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t}$$

$$\frac{\partial \hat{m}}{\partial t} = -\gamma \hat{m} \times \left(-\frac{\partial E}{\partial \vec{M}} + a_J (\hat{m} \times \hat{p}) + b_J \hat{p} \right) + \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t}$$

$\hat{p}$ - represents the magnetization direction of the reference layer in the STT case
or
average spin direction of electrons diffusing into the magnetic layer in SOT case

$|H_A| \ll |H \sin \theta_H|$

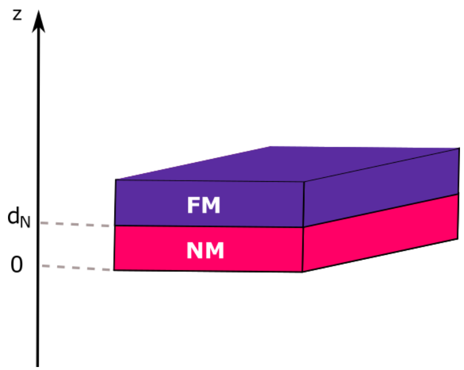
$\varphi_0 = \varphi_H$

PHENOMENOLOGICAL DESCRIPTION OF SOTs

MOTIVATION:

J. Kim, J. Sinha, S. Mitani, M. Hayashi, S. Takahashi, S. Maekawa, M. Yamanouchi, and H. Ohno, ***Anomalous temperature dependence of current-induced torques in CoFeB/MgO heterostructures with Ta-based underlayers,*** PHYSICAL REVIEW B **89**, 174424 (2014)

DRIFT-DIFFUSION FORMALISM



$$\vec{J}_N = J_N \hat{x}$$

$$Q_{iz}(z) = -\frac{1}{2|e|\rho_N}\frac{\partial}{\partial z}\delta\mu_i + \theta_{SH}(\hat{e}_i \times \vec{J}_N)_z$$

$$\delta\vec{\mu}(z) = \vec{A}\exp[-z/\lambda_N] + \vec{B}\exp[z/\lambda_N]$$

$$Q_{iz}^{N|F} = -\frac{1}{2|e|}\mathrm{Re}[G_{\mathrm{MIX}}](\hat{m}\times\hat{m}\times\delta\vec{\mu}(z=d))_i - \frac{1}{2|e|}\mathrm{Im}[G_{\mathrm{MIX}}](\hat{m}\times\delta\vec{\mu}(z=d))_i,$$

BOUNDARY CONDITIONS:

$$Q_{iz}(z=0) = 0,$$

$$Q_{iz}(z=d) = Q_{iz}^{N|F}$$

$$\Delta H_j = -\frac{\hbar}{2|e|t_F M_S}(\hat{m}\times\vec{Q}_z^{N|F})_j$$

$$\Delta H_x = -\theta_{SH}\frac{\hbar}{2|e|}\frac{J_N}{M_S t_F}\left(1-\frac{1}{\cosh(d/\lambda_N)}\right)\left[\frac{(1+g_r)g_r+g_i^2}{(1+g_r)^2+g_i^2}\right]\mathrm{sgn}(\hat{m}\times\hat{e}_y)_x,$$

$$\Delta H_y = -\theta_{SH}\frac{\hbar}{2|e|}\frac{J_N}{M_S t_F}\left(1-\frac{1}{\cosh(d/\lambda_N)}\right)\left[\frac{g_i}{(1+g_r)^2+g_i^2}\right]$$

$$g_r = \mathrm{Re}[G_{\mathrm{MIX}}]\rho_N\lambda_N\coth(\tfrac{d}{\lambda_N})$$
$$g_i = \mathrm{Im}[G_{\mathrm{MIX}}]\rho_N\lambda_N\coth(\tfrac{d}{\lambda_N})$$

PHENOMENOLOGICAL DESCRIPTION OF SOTs

❑ MOTIVATION:

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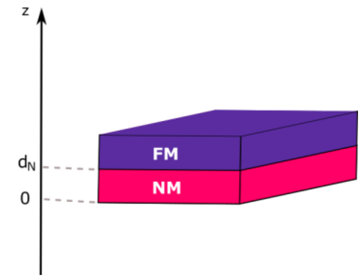


▪ RESULTS

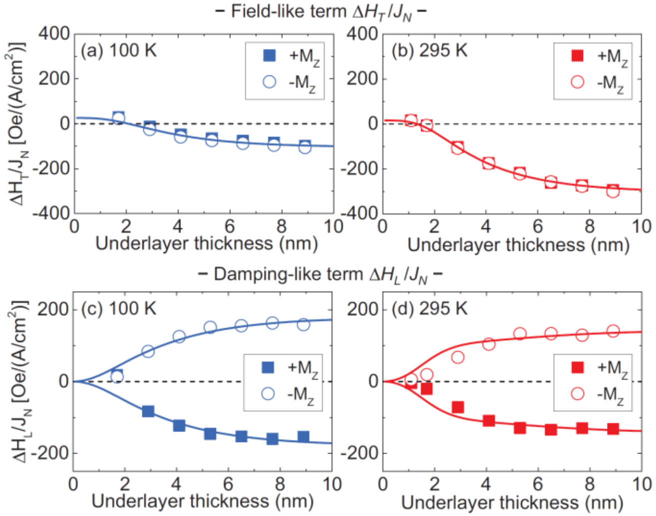


FIG. 6. (Color online) Underlayer thickness dependence of (a) and (b) $\Delta H_T/J_N$ and (c) and (d) $\Delta H_L/J_N$ for the TaN underlayer film. The measurement temperature is 100 K for (a) and (c) and RT (~295 K) for (b) and (d). Solid lines are calculated using Eqs. (7) and (8) with the following parameters: $M_S = 1200$ emu/cm^3, $\rho_N = 395$ $\mu\Omega$·cm, $\lambda_N = 2.5$ nm, $\theta_{SH} = -0.08$, and the real and imaginary parts of the spin mixing conductance (G_{MIX}), which are shown by the solid symbols in Fig. 7(a).

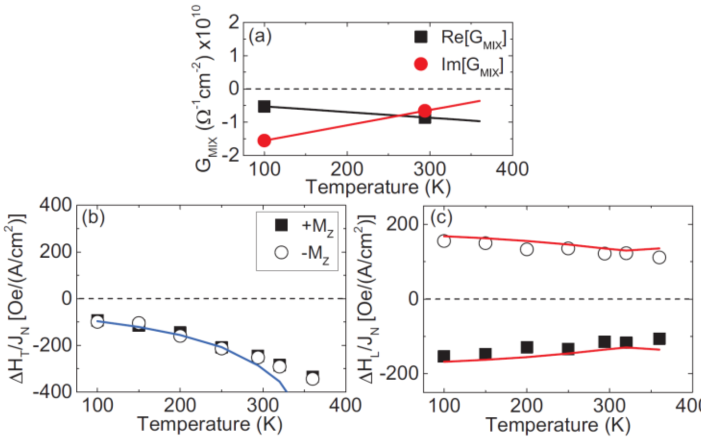
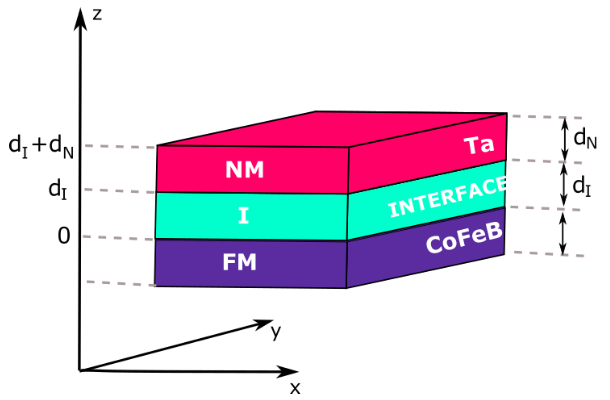


FIG. 7. (Color online) (a) Temperature dependence of the real and imaginary parts of the spin mixing conductance (G_{MIX}) obtained by the fitting shown in Fig. 6. Linear fit to the data (symbols) is shown by the solid line. Temperature dependence of (b) $\Delta H_T/J_N$ and (c) $\Delta H_L/J_N$ for the TaN underlayer film with $d \sim 8.9$ nm. Solid lines are calculated using Eqs. (7) and (8) with the following parameters: $M_S =$ 1200 emu/cm^3, $\rho_N = 395$ $\mu\Omega$·cm, $\lambda_N = 2.5$ nm, $\theta_{SH} = -0.08$, and the interpolated real and imaginary parts of G_{MIX} shown by the solid lines in (a).

PHENOMENOLOGICAL DESCRIPTION OF SOTs

❑ EFFECTS ON INTERFACE



The NM layer and the FM layer affect the electronic strucyure of each other close to the interface

- the magnetic proximity effect can induce a magnetic moment in the NM layer (the material with strong SOC)
- the spin-orbit proximity effect can induce spin-orbit effects in the FM layer
- lattice mismatch between materials: thickness fluctuations, formation of dislocation
- ……

✓ **spin backflow** (even from a „well-ordered" interface)

✓ **spin memory loss**
due to intermixing and disorder at the NM|FM interface

PHENOMENOLOGICAL DESCRIPTION OF SOTs

❑ EFFECTS ON INTERFACE: TRILAYER SPIN DIFFUSION MODEL

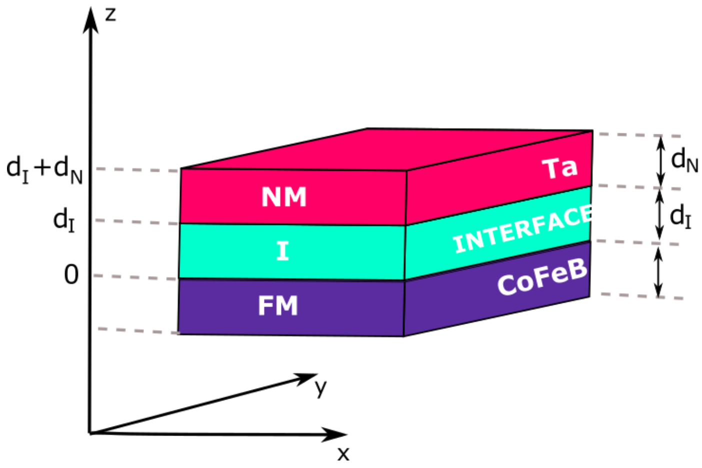
$$\vec{j}_s^N(z) = -\frac{1}{2|e|\rho_N}\frac{\partial \vec{\mu}_s^N(z)}{\partial z} - \theta_{SH} J_N \hat{y}$$

$$\frac{\partial^2 \vec{\mu}_s^N(z)}{\partial z^2} = \frac{\vec{\mu}_s^N(z)}{\lambda_{s,N}^2} \qquad \vec{\mu}_s^N(z) = \vec{C}\exp(-\frac{z}{\lambda_N}) + \vec{D}\exp(\frac{z}{\lambda_N})$$

$$\vec{j}_s^I(z) = -\frac{1}{2|e|\rho_I}\frac{\partial \vec{\mu}_s^I(z)}{\partial z}$$

$$\frac{\partial^2 \vec{\mu}_s^I(z)}{\partial z^2} = \frac{\vec{\mu}_s^I(z)}{\lambda_{s,I}^2} \qquad \vec{\mu}_s^I(z) = \vec{A}\exp(-\frac{z}{\lambda_I}) + \vec{B}\exp(\frac{z}{\lambda_I})$$

$$|e|\vec{j}_s^{F|I} = G_r \hat{m}\times\hat{m}\times\vec{\mu}_s^I(0) + G_i \hat{m}\times\vec{\mu}_s^I(0)$$



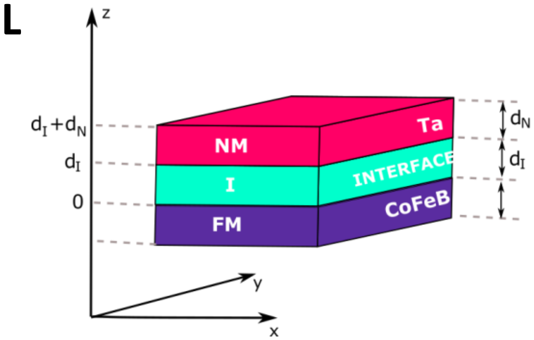
BOUNDARY CONDITIONS:

$$\vec{j}_s^N(z=d_I) = \vec{j}_s^I(z=d_I)\,,$$
$$\vec{j}_s^I(z=0) = \vec{j}_s^{F|I}\,,$$
$$\vec{j}_s^N(z=d_I+d_N) = 0\,,$$
$$\vec{\mu}_s^I(z=d_I) = \vec{\mu}_s^N(z=d_I)$$

$$\vec{\mu}_s^I(0) = 2|e|\theta_{SH}J_N\hat{y}\frac{\tanh(\frac{d_N}{2\lambda_N})\mathrm{csch}(\frac{d_I}{\lambda_I})}{\frac{1}{\rho_I\lambda_I}\coth(\frac{d_N}{\lambda_N}) + \frac{1}{\rho_N\lambda_N}\coth(\frac{d_I}{\lambda_I})} + 2|e|\vec{j}_s^{F|I}\frac{\coth(\frac{d_N}{\lambda_N})\coth(\frac{d_I}{\lambda_I}) + \frac{\rho_I\lambda_I}{\rho_N\lambda_N}}{\frac{1}{\rho_I\lambda_I}\coth(\frac{d_N}{\lambda_N}) + \frac{1}{\rho_N\lambda_N}\coth(\frac{d_I}{\lambda_I})}$$

PHENOMENOLOGICAL DESCRIPTION OF SOTs

EFFECTS ON INTERFACE: TRILAYER SPIN DIFFUSION MODEL



DAMPING-LIKE AND FIELD-LIKE TORQUE EFFICIENCY:

$$\xi_{DL} \equiv \frac{2|e|\mu_0 M_S t_F}{\hbar J_N}\Delta H_L \qquad \xi_{FL} \equiv \frac{2|e|\mu_0 M_S t_F}{\hbar J_N}\Delta H_T$$

$$\xi_{DL} = -\theta_{SH}\frac{\tanh(\frac{d_N}{2\lambda_N})\text{csch}(\frac{d_I}{\lambda_I})}{\coth(\frac{d_N}{\lambda_N})\coth(\frac{d_I}{\lambda_I}) + \frac{\rho_I\lambda_I}{\rho_N\lambda_N}}\frac{g_r\,(1+g_r)+g_i^2}{(1+g_r)^2+g_i^2}m_z$$

$$\xi_{FL} = -\theta_{SH}\frac{\tanh(\frac{d_N}{2\lambda_N})\text{csch}(\frac{d_I}{\lambda_I})}{\coth(\frac{d_N}{\lambda_N})\coth(\frac{d_I}{\lambda_I}) + \frac{\rho_I\lambda_I}{\rho_N\lambda_N}}\frac{g_i}{(1+g_r)^2+g_i^2}$$

$$g_{r,i} = 2G_{r,i}\frac{\coth(\frac{d_N}{\lambda_N})\coth(\frac{d_I}{\lambda_I}) + \frac{\rho_I\lambda_I}{\rho_N\lambda_N}}{\frac{1}{\rho_I\lambda_I}\coth(\frac{d_N}{\lambda_N}) + \frac{1}{\rho_N\lambda_N}\coth(\frac{d_I}{\lambda_I})}$$

$$G_i = \text{Im}\{\text{G}_{\text{mix}}\},\ G_r = \text{Re}\{\text{G}_{\text{mix}}\}$$

- **WITHOUT INTERFACE LAYER**

$$\xi_{DL} = -\theta_{SH}\left(1-\frac{1}{\cosh(\frac{d}{\lambda_N})}\right)\frac{(1+g_r)g_r+g_i^2}{(1+g_r)^2+g_i^2}$$

$$\xi_{FL} = -\theta_{SH}\left(1-\frac{1}{\cosh(\frac{d}{\lambda_N})}\right)\frac{g_i}{(1+g_r)^2+g_i^2}$$

$$g_{r,i} = G_{r,i}\rho_N\lambda_N\coth(d/\lambda_N)$$

PHENOMENOLOGICAL DESCRIPTION OF SOTs

❑ EFFECTS ON INTERFACE: TRILAYER SPIN DIFFUSION MODEL – RESULTS

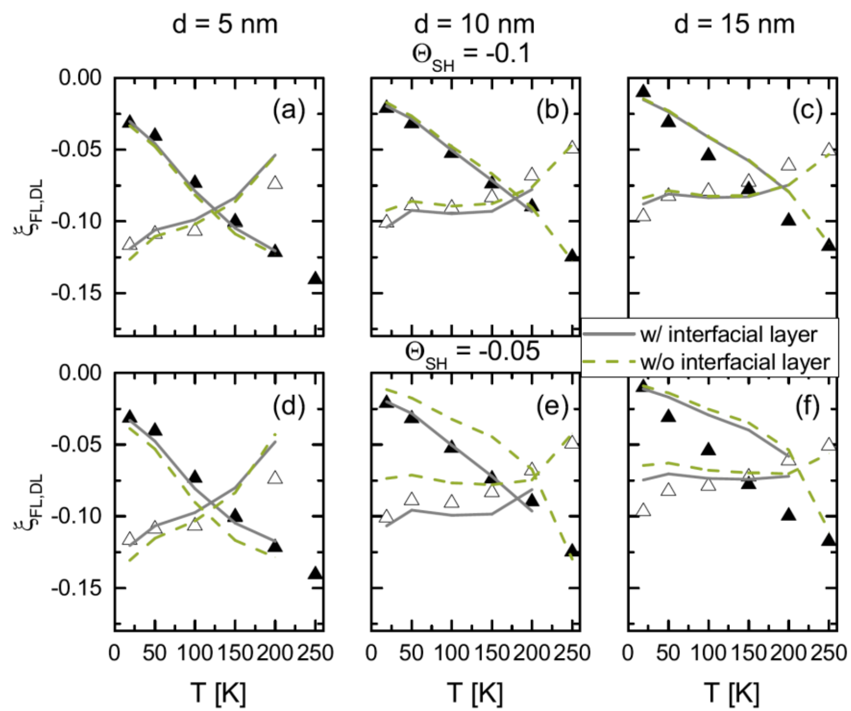
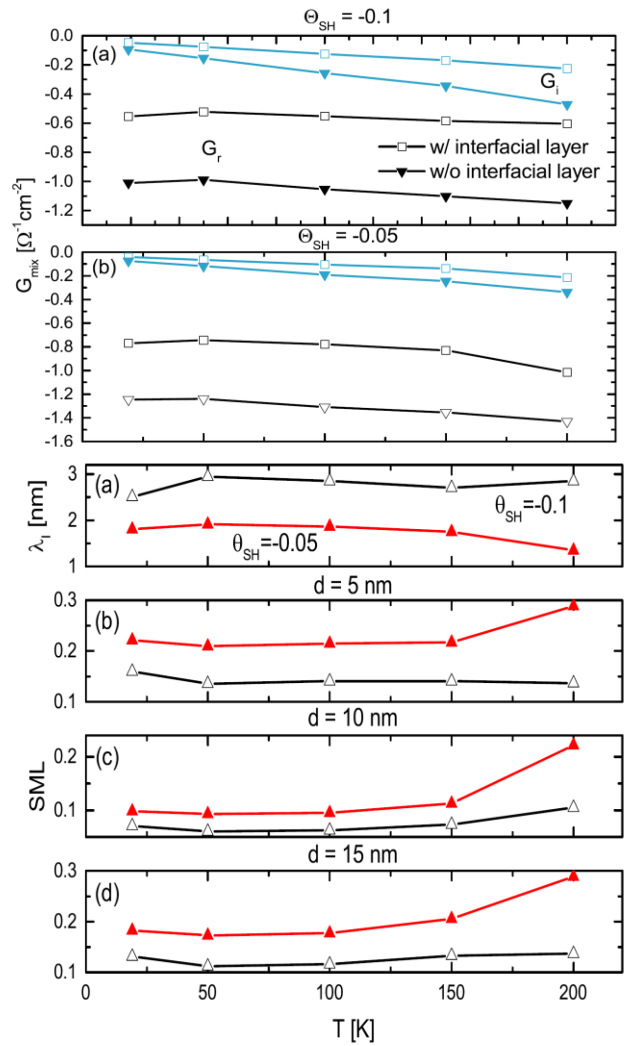


FIG. 6: Spin-torque efficiencies with theoretical fit for the case of additional and no interfacial layer in the model for indicated values of spin Hall angle θ_{SH} and indicated values of Ta underlayer thickness for Ta spin diffusion length $\lambda_N = 2.5$ nm.



TRILAYER SPIN DIFFUSION MODEL : SUMMARY

- **Im[G_{MIX}] comparable in size to its real counterpart**
  - ✓ large angle rotation of the spin direction of spin current
  - ✓ presence of spin Hall torque

- **Negative Re[G_{MIX}] !!!**
  - ✓ model failure
  - ✓ poor band matching between NM and the FM layers
  - ✓ flow of spin current from the FM layer to the NM layer

AHE or SHE in FM layer should be checked !

ANOMALOUS HALL EFFECT IN Ta|CoFeB

JOURNAL OF APPLIED PHYSICS **113**, 17C717 (2013)

CrossMark

The anomalous Hall effect in the perpendicular Ta/CoFeB/MgO thin films

S. B. Wu,[1,2] T. Zhu,[1,a)] X. F. Yang,[2] and S. Chen[2]
[1]*Institute of Physics and Beijing National Laboratory for Condensed Matter Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*
[2]*Department of Electronic Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China*

(Presented 17 January 2013; received 2 November 2012; accepted 20 December 2012; published online 25 March 2013)

The anomalous Hall effect (AHE) in the perpendicular Ta/CoFeB/MgO thin film has been investigated. Between the AHE coefficient (R_S) and longitudinal resistivity (ρ_{xx}), a linear behavior of R_S/ρ_{xx} versus ρ_{xx} can be found. Moreover, the conductivity of the film is about 5×10^3 S/cm, which suggests that AHE in the Ta/CoFeB/MgO film be dominated by the intrinsic or scattering-independent mechanism. [http://dx.doi.org/10.1063/1.4796192]

APPLIED PHYSICS LETTERS **104**, 202404 (2014)

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Giant linear anomalous Hall effect in the perpendicular CoFeB thin films

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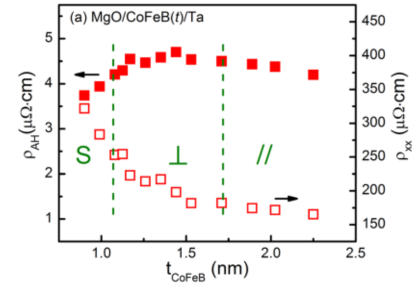
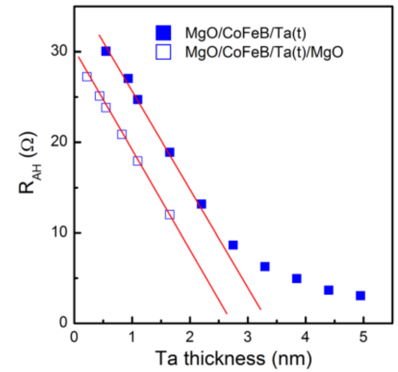
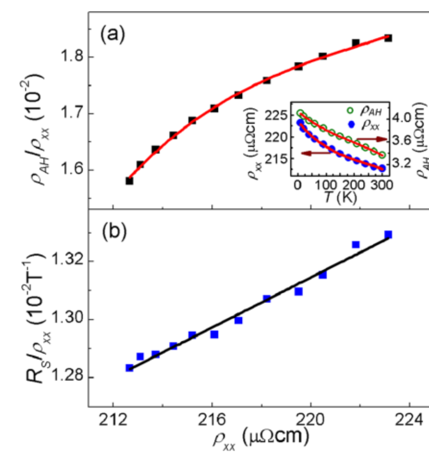
(Received 26 March 2014; accepted 7 May 2014; published online 20 May 2014)

Giant anomalous Hall effect in a perpendicular CoFeB thin film has been reported, which satisfies large anomalous Hall resistivity and low switching field at same time. The largest sensitivity of linear anomalous Hall effect reaches 2376 Ω/kOe, which is 21 times larger than that of the best semiconductor. Our results suggest the low cost MgO/CoFeB/Ta thin film can be a potential candidate for highly sensitive magnetic field detecting.
[http://dx.doi.org/10.1063/1.4878538]







TRILAYER SPIN DIFFUSION MODEL : SUMMARY

- **Im[G_{MIX}] comparable in size to its real counterpart**
  - ✓ large angle rotation of the spin direction of spin current
  - ✓ presence of spin Hall torque

- **Negative Re[G_{MIX}] !!!**
  - ✓ model failure
  - ✓ poor band matching between NM and the FM layers
  - ✓ flow of spin current from the FM layer to the NM layer

AHE or SHE in FM layer should be checked !

➡ **FURTHER MODIFICATION OF TRILAYER SPIN DIFFUSION MODEL**

$$\vec{j}_s^N(z) = -\frac{1}{2|e|\rho_N}\frac{\partial \vec{\mu}_s^N(z)}{\partial z} - \theta_{SH} J_N \hat{y}$$

$$\frac{\partial^2 \vec{\mu}_s^N(z)}{\partial z^2} = \frac{\vec{\mu}_s^N(z)}{\lambda_{s,N}^2} \qquad \vec{\mu}_s^N(z) = \vec{C}\exp(-\frac{z}{\lambda_N}) + \vec{D}\exp(\frac{z}{\lambda_N})$$

$$\vec{j}_s^I(z) = -\frac{1}{2|e|\rho_I}\frac{\partial \vec{\mu}_s^I(z)}{\partial z} \boxed{-(\theta_{SH} + \Delta\theta_{SH}) J_{Iy} ???}$$

SOC at the interface !

$$\frac{\partial^2 \vec{\mu}_s^I(z)}{\partial z^2} = \frac{\vec{\mu}_s^I(z)}{\lambda_{s,I}^2} \qquad \vec{\mu}_s^I(z) = \vec{A}\exp(-\frac{z}{\lambda_I}) + \vec{B}\exp(\frac{z}{\lambda_I})$$

$$|e|\vec{j}_s^{F|I} = G_r \hat{m} \times \hat{m} \times \vec{\mu}_s^I(0) + G_i \hat{m} \times \vec{\mu}_s^I(0)$$

SOC in the FM ?

ANOTHER APPROACHES

Spin Transport at Interfaces with Spin-Orbit Coupling: Phenomenology

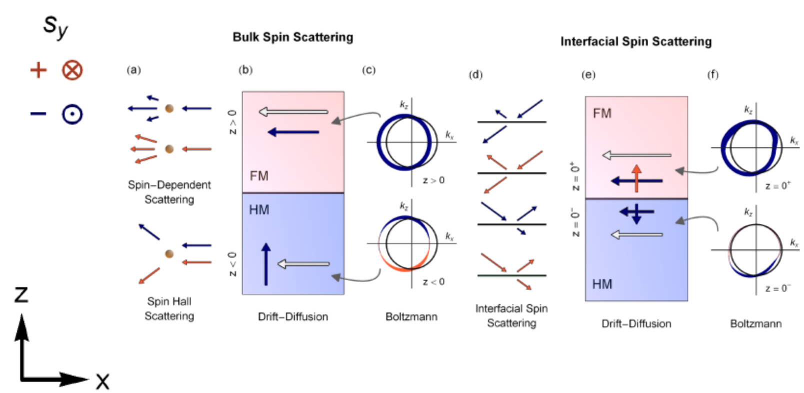
V. P. Amin[1,2,*] and M. D. Stiles[2]

$$\mathbf{j}_s(z) = \frac{\sigma^{\mathrm{FM}}}{e} P \hat{\mathbf{m}} \frac{\partial \mu_c(z)}{\partial z} - \frac{\sigma^{\mathrm{FM}}}{e} \frac{\partial \boldsymbol{\mu}_s(z)}{\partial z}$$

$$\frac{1}{eN_s^{\mathrm{FM}}} \frac{\partial \mathbf{j}_s(z)}{\partial z} = -\frac{1}{\tau_{\mathrm{sf}}^{\mathrm{FM}}} \boldsymbol{\mu}_s(z) - \frac{1}{\tau_{\mathrm{ex}}} \boldsymbol{\mu}_s(z) \times \hat{\mathbf{m}} - \frac{1}{\tau_{\mathrm{dp}}} \hat{\mathbf{m}} \times \boldsymbol{\mu}_s(z) \times \hat{\mathbf{m}}$$

$$\mathbf{j}_s(z) = -\frac{\sigma^{\mathrm{HM}}}{e} \frac{\partial \boldsymbol{\mu}_s(z)}{\partial z} + \mathbf{j}_s^{\mathrm{sH}} \qquad \mathbf{j}_s^{\mathrm{sH}} = \sigma_{\mathrm{sH}} \mathbf{E} \times \hat{\mathbf{z}}$$

$$\frac{1}{eN_s^{\mathrm{HM}}} \frac{\partial \mathbf{j}_s(z)}{\partial z} = -\frac{1}{\tau_{\mathrm{sf}}^{\mathrm{HM}}} \boldsymbol{\mu}_s(z).$$



BOUNDARY CONDITIONS:

$$\mathbf{j}_\perp(0^-) = \mathbf{G}_R \boldsymbol{\mu}_\perp(0^-) + \mathbf{j}_\perp^{\mathrm{E}}(0^-) \qquad \mathbf{j}_\perp^{\mathrm{E}}(0^-) = \boldsymbol{\sigma}(\hat{\mathbf{m}})\tilde{E}$$

$$\mathbf{j}_\perp(0^+) = \boldsymbol{\Gamma}^{\mathrm{FM}} \boldsymbol{\mu}_\perp(0^-) + \mathbf{j}_\perp^{\mathrm{E}}(0^+) \qquad \mathbf{j}_\perp^{\mathrm{E}}(0^+) = \boldsymbol{\gamma}^{\mathrm{FM}}(\hat{\mathbf{m}})\tilde{E}$$

$$\boldsymbol{\tau}^{\mathrm{mag}} = (\mathbf{G}_R - \boldsymbol{\Gamma}^{\mathrm{FM}}) \boldsymbol{\mu}_\perp(0^-) + \boldsymbol{\tau}^{\mathrm{E}} \qquad \boldsymbol{\tau}^{\mathrm{E}} = \boldsymbol{\gamma}^{\mathrm{mag}}(\hat{\mathbf{m}})\tilde{E}$$

spin currents generated by interfacial spin-orbit scattering

additional spin-torque due to spin polarization that arises at the interface

the torque on magnetization at the interface

$$V(\mathbf{r}) = \frac{\hbar^2 k_F}{m} \delta(z) \left(u_0 + u_{\mathrm{ex}} \boldsymbol{\sigma} \cdot \hat{\mathbf{m}} + u_R \boldsymbol{\sigma} \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{z}}) \right)$$

THE RASHBA MODEL OF SOC

- conductance matrix - $\mathbf{G}_R = \begin{pmatrix} \mathrm{Re}[G_{\uparrow\downarrow}] & -\mathrm{Im}[G_{\uparrow\downarrow}] \\ \mathrm{Im}[G_{\uparrow\downarrow}] & \mathrm{Re}[G_{\uparrow\downarrow}] \end{pmatrix}$
- torkance tensor - $\boldsymbol{\Gamma}^{\mathrm{FM}}$
- conductivity tensor - $\boldsymbol{\sigma}(\hat{\mathbf{m}})$
- torkivity tensors - $\boldsymbol{\gamma}^{\mathrm{mag}}(\hat{\mathbf{m}})$, $\boldsymbol{\gamma}^{\mathrm{FM}}(\hat{\mathbf{m}})$.

ANOTHER APPROACHES

PRL **114,** 126602 (2015) PHYSICAL REVIEW LETTERS week ending 27 MARCH 2015

Spin Pumping in the Presence of Spin-Orbit Coupling

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(Received 20 October 2014; published 26 March 2015)

Spin pumping and related phenomena have been observed recently in heavy metals and topological insulators, where the spin-orbit coupling plays an essential role. We have developed a spin-pumping formalism that explicitly includes the spin-orbit coupling at interfaces and disorder in the layers. Spin pumping across an interface with spin-orbit coupling and the attendant backflow are treated on an equal footing. We resolve some long-standing issues on the conflicting conclusions about the spin-diffusion length for Pt, and the origin of spin-memory loss at interfaces with heavy metals. In addition, we predict some heretofore unanticipated spin-pumping phenomena.

DOI: 10.1103/PhysRevLett.114.126602 PACS numbers: 72.25.Mk, 73.40.-c

- ✓ spin pumping conductivity is given by retarded and advanced Green functions

$$\mathbf{j}^s = \frac{\hbar}{4\pi}\left(g_r \mathbf{m}\times\frac{d\mathbf{m}}{dt} - g_i\frac{d\mathbf{m}}{dt}\right)$$

$$\mathbf{j}_i(\mathbf{r},t) = \frac{\hbar}{4\pi}\left[\Gamma_i^{re}(r)\mathbf{m}\times\frac{d\mathbf{m}}{dt} - \Gamma_i^{im}(r)\frac{d\mathbf{m}}{dt}\right]$$

$$\Gamma_i(\mathbf{r}) = \Gamma_i^{re}(\mathbf{r}) + i\Gamma_i^{im}(\mathbf{r}) \qquad \Gamma_i(\mathbf{r}) = \frac{J_{\mathrm{ex}}\hbar^2}{m_e}\int_{\mathrm{FM}} d^3r'\, g_\uparrow^R(\mathbf{r},\mathbf{r}')\overleftrightarrow{\partial}_i g_\downarrow^A(\mathbf{r}',\mathbf{r})$$

- ✓ disoredrs included explicitly
- ✓ SOC at the interface
- ✓ backflow current

ANOTHER APPROACHES

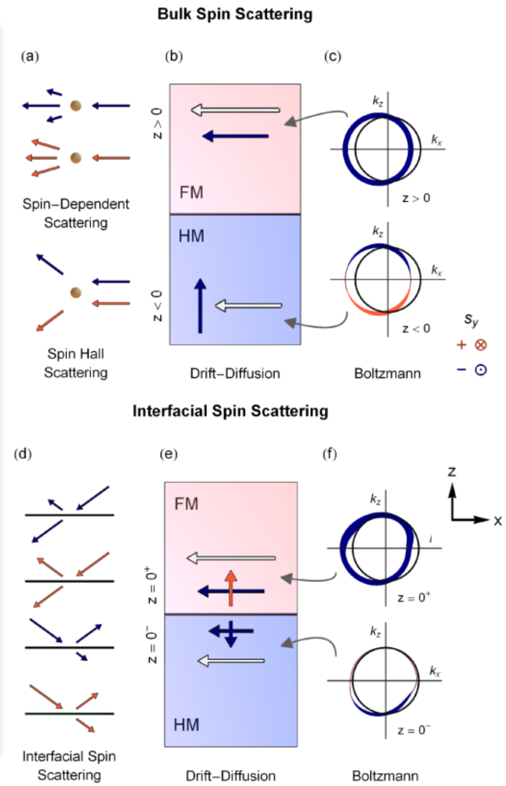
Spin Transport at Interfaces with Spin-Orbit Coupling: Phenomenology

V. P. Amin[1,2,*] and M. D. Stiles[2]

[1]Maryland NanoCenter, University of Maryland, College Park, MD 20742
[2]Center for Nanoscale Science and Technology, National Institute of Standards and Technology, Gaithersburg, Maryland 20899, USA

(Dated: April 25, 2016)

Spin transport remains poorly understood in multilayer systems with interfacial spin-orbit coupling. While the important consequences of interfacial spin-orbit coupling can be captured by a spin-dependent Boltzmann equation, currently they cannot be captured by drift-diffusion models, which are the primary tools used for analyzing experiments. Here we present boundary conditions suitable for drift-diffusion models that capture the phenomenology of spin-orbit torques at interfaces. We compare solutions of the drift-diffusion equations using these boundary conditions to solutions of the spin-dependent Boltzmann equation for a heavy metal/ferromagnet bilayer. We find that the drift-diffusion equations predict spin torques in quantitative agreement with the Boltzmann equation and allow for a much simpler interpretation of the results. A key feature of these boundary conditions is their ability to capture the scattering that carriers experience while flowing along an interface. In the presence of interfacial spin-orbit coupling, this scattering generates spin currents that flow away from the interface. In heavy metal/ferromagnet bilayers, these spin currents provide an important mechanism for the creation of damping-like and field-like torques; they also lead to possible reinterpretations of experiments in which interfacial contributions to spin torques are thought to be suppressed.



$$\mathbf{j}_s(z) = \frac{\sigma^{\rm FM}}{e} P\hat{\mathbf{m}}\frac{\partial \mu_c(z)}{\partial z} - \frac{\sigma^{\rm FM}}{e}\frac{\partial \boldsymbol{\mu}_s(z)}{\partial z}$$

$$\frac{1}{eN_s^{\rm FM}}\frac{\partial \mathbf{j}_s(z)}{\partial z} = -\frac{1}{\tau_{\rm sf}^{\rm FM}}\boldsymbol{\mu}_s(z) - \frac{1}{\tau_{\rm ex}}\boldsymbol{\mu}_s(z)\times\hat{\mathbf{m}} - \frac{1}{\tau_{\rm dp}}\hat{\mathbf{m}}\times\boldsymbol{\mu}_s(z)\times\hat{\mathbf{m}}$$

$$\mathbf{j}_s(z) = -\frac{\sigma^{\rm HM}}{e}\frac{\partial \boldsymbol{\mu}_s(z)}{\partial z} + \mathbf{j}_s^{\rm sH} \qquad \mathbf{j}_s^{\rm sH} = \sigma_{\rm sH}\mathbf{E}\times\hat{\mathbf{z}}$$

$$\frac{1}{eN_s^{\rm HM}}\frac{\partial \mathbf{j}_s(z)}{\partial z} = -\frac{1}{\tau_{\rm sf}^{\rm HM}}\boldsymbol{\mu}_s(z).$$

$$\mathbf{j}_\perp(0^-) = \mathbf{G}_R\boldsymbol{\mu}_\perp(0^-) + \mathbf{j}_\perp^{\rm E}(0^-) \qquad \mathbf{j}_\perp^{\rm E}(0^-) = \boldsymbol{\sigma}(\hat{\mathbf{m}})\tilde{E}$$

$$\mathbf{j}_\perp(0^+) = \boldsymbol{\Gamma}^{\rm FM}\boldsymbol{\mu}_\perp(0^-) + \mathbf{j}_\perp^{\rm E}(0^+) \qquad \mathbf{j}_\perp^{\rm E}(0^+) = \boldsymbol{\gamma}^{\rm FM}(\hat{\mathbf{m}})\tilde{E}$$

$$\boldsymbol{\tau}^{\rm mag} = (\mathbf{G}_R - \boldsymbol{\Gamma}^{\rm FM})\boldsymbol{\mu}_\perp(0^-) + \boldsymbol{\tau}^{\rm E} \qquad \boldsymbol{\tau}^{\rm E} = \boldsymbol{\gamma}^{\rm mag}(\hat{\mathbf{m}})\tilde{E}$$

$$\mathbf{G}_R = \begin{pmatrix} {\rm Re}[G_{\uparrow\downarrow}] & -{\rm Im}[G_{\uparrow\downarrow}] \\ {\rm Im}[G_{\uparrow\downarrow}] & {\rm Re}[G_{\uparrow\downarrow}] \end{pmatrix}$$

$$V(\mathbf{r}) = \frac{\hbar^2 k_F}{m}\delta(z)\big(u_0 + u_{\rm ex}\boldsymbol{\sigma}\cdot\hat{\mathbf{m}} + u_R\boldsymbol{\sigma}\cdot(\hat{\mathbf{k}}\times\hat{\mathbf{z}})\big)$$